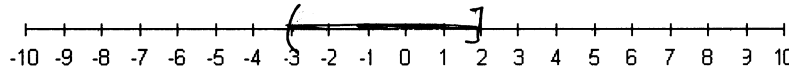


Math 111

Final Exam Name KEY

Section _____ Date _____

1. (a) Graph the solution set specified by the given statement. 2 points
 $\{x \mid x > -3 \text{ and } x \leq 2\}$



- (b) Express the final solution set from part (a) in simplest interval format.

Interval(s): $(-3, 2]$

2 points

2. Write in simplest form with only positive exponents.

$$\begin{aligned} & \left(2a^{\frac{2}{3}}b^{-4}\right)^3 \left(-3a^{-\frac{1}{2}}b^{\frac{3}{2}}\right)^2 \\ & (8a^2b^{-12})(9a^{-1}b^3) \\ & 72ab^{-9} \\ & \frac{72a}{b^9} \end{aligned}$$

2. $\frac{72a}{b^9}$

4 points

3. Perform the indicated operations and simplify where possible. Show all work to receive credit.

$$\begin{aligned} & \frac{3y}{y^2-7y+10} - \frac{2y}{y^2-8y+15} \\ & \frac{3y}{(y-2)(y-5)} - \frac{2y}{(y-3)(y-5)} \\ & \frac{3y(y-3) - 2y(y-2)}{(y-2)(y-3)(y-5)} \end{aligned}$$

$$\begin{aligned} & \frac{3y^2 - 9y - 2y^2 + 4y}{(y-2)(y-3)(y-5)} \\ & \frac{y^2 - 5y}{(y-2)(y-3)(y-5)} = \frac{y(y-5)}{(y-2)(y-3)(y-5)} \\ & \frac{y}{(y-2)(y-3)} \end{aligned}$$

3. $\frac{y}{(y-2)(y-3)}$ or $\frac{y}{y^2-5y+6}$

3 points

4. Write an equation for a function that has the shape of $y=|x|$, but is shifted left 4 units and up 3 units.

4. $y = |x+4| + 3$

2 points

5. The points $(-2, 3)$ and $(4, -5)$ are the endpoints of the diameter of a circle. Find the length of the radius of the circle.

$$d = \sqrt{(-2-4)^2 + (3-(-5))^2}$$

$$d = \sqrt{(-6)^2 + (8)^2} = \sqrt{36+64} = \sqrt{100}$$

$$d = 10$$

$$r = \frac{10}{2} = 5$$

5. 5
3 points

6. Find the equations of the line that passes through the point $(-1, 5)$ and is perpendicular to the line $3x - 4y = -8$. Write in slope-intercept form.

$$3x - 4y = -8$$

$$-4y = -3x - 8$$

$$y = \frac{3}{4}x + 2$$

$$m = \frac{3}{4}$$

$$y - 5 = -\frac{4}{3}(x + 1)$$

$$3y - 15 = -4x - 4$$

$$3y = -4x + 11$$

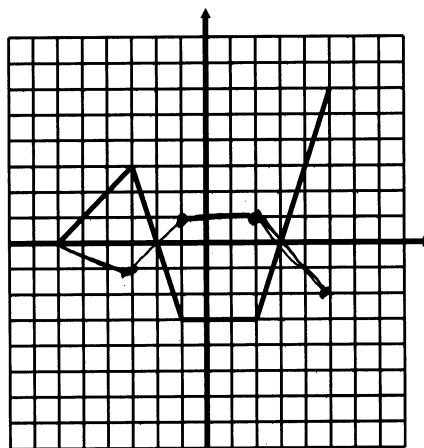
$$y = -\frac{4}{3}x + \frac{11}{3}$$

$$(-1, 5) \quad m = -\frac{4}{3}$$

6. $y = -\frac{4}{3}x + \frac{11}{3}$
4 points

7. A graph of $y = f(x)$ is given below. No formula for f is given.

$$\text{Graph } y = -\frac{1}{3}f(x).$$



4 points

8. Solve the inequality $|y + 5| \geq 2$ giving the solution in interval notation.

$$y + 5 \geq 2 \quad \text{or} \quad y + 5 \leq -2$$

$$y \geq -3 \quad \text{or} \quad y \leq -7$$

8. $(-\infty, -7] \cup [-3, \infty)$
3 points

9. Solve for a: $\frac{5}{a-4} - \frac{3}{a-1} = \frac{a+1}{a-4}$

$$5(a-1) - 3(a-4) = (a-1)(a+1)$$

$$5a - 5 - 3a + 12 = a^2 - 1$$

$$2a + 7 = a^2 - 1$$

$$a^2 - 2a - 8 = 0$$

$$(a-4)(a+2) = 0$$

$$a = 4 \quad a = -2$$

$$a \neq 4$$

10. Consider the function $g(x) = x^2 - 15x + 36$ and find:

a) the zeros of $g(x)$

$$(x-12)(x-3) = 0$$

$$x = 12 \quad x = 3$$

a) 12, 3

2 points

b) the vertex of the graph of $g(x)$

$$x = \frac{-b}{2a} = \frac{15}{2}$$

$$y = g\left(\frac{15}{2}\right) = \left(\frac{15}{2}\right)^2 - 15\left(\frac{15}{2}\right) + 36 = \frac{-81}{4}$$

b) (7.5, -20.25)

2 points

c) the range of $g(x)$ (in interval notation): c) $[-20.25, \infty)$

2 points

11. Solve for b: $-2(b+3)^2 = 40$

$$(b+3)^2 = -20$$

$$b+3 = \pm\sqrt{-20}$$

$$b+3 = \pm 2i\sqrt{5}$$

$$b = -3 \pm 2i\sqrt{5}$$

11. $-3 \pm 2i\sqrt{5}$

3 points

12. A ball is thrown vertically upward with an initial speed of 48 ft/s. Its height, in feet, after t seconds is given by $h(t) = -16t^2 + 48t$. Find the maximum height of the ball and the time when it reaches that height.

$$\frac{-b}{2a} = \frac{-48}{-32} = 1.5$$

$$h(1.5) = -16(1.5)^2 + 48(1.5) = 36$$

$$\text{time} = 1.5 \text{ Sec}$$

$$12. \underline{\text{height} = 36 \text{ ft}}$$

4 points

13. Use division to find the quotient $Q(x)$ and the remainder $R(x)$, and express $P(x)$ in the form $d(x) \cdot Q(x) + R(x)$.

$$P(x) = x^3 - 2x^2 + x - 6$$

$$d(x) = x + 3$$

$$\begin{array}{r} x^2 - 5x + 16 \\ x+3 \overline{) x^3 - 2x^2 + x - 6} \\ \underline{x^3 + 3x^2} \\ -5x^2 + x \\ \underline{-5x^2 - 15x} \\ 16x - 6 \\ \underline{16x + 48} \\ -54 \end{array}$$

$$13. \underline{P(x) = (x+3)(x^2 - 5x + 16) + -54}$$

3 points

14. Find a polynomial of degree 4 with -2 as a zero of multiplicity 2 and 0 and 3 as zeros of multiplicity 1 and write as $P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} \dots + a_1 x + a_0$.

$$P(x) = x(x-3)(x+2)^2$$

$$P(x) = x(x-3)(x^2 + 4x + 4)$$

$$P(x) = (x^2 - 3x)(x^2 + 4x + 4)$$

$$P(x) = x^4 + 4x^3 + 4x^2 - 3x^3 - 12x^2 - 12x$$

$$P(x) = x^4 + x^3 - 8x^2 - 12x$$

$$14. \underline{P(x) = x^4 + x^3 - 8x^2 - 12x}$$

4 points

15. Make a graph of $f(x) = \frac{6}{(x-2)^2}$.

(a) Label all the asymptotes and y-intercept.

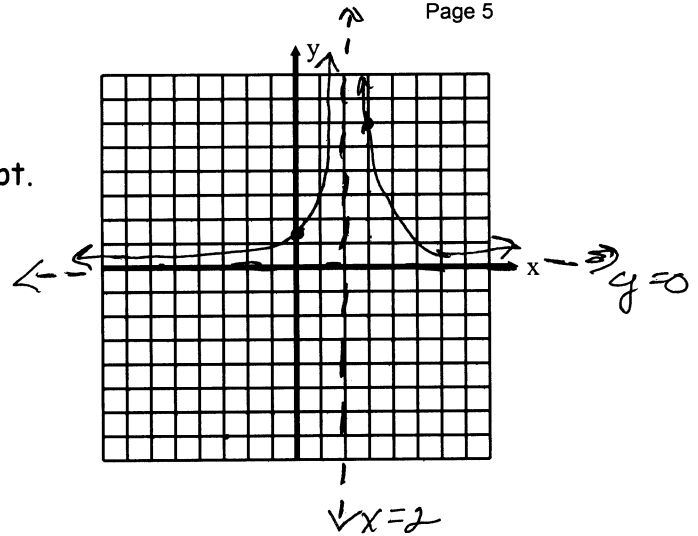
V.A. $x=2$ y-int $(0, \frac{3}{2})$ 3 points
H.A. $y=0$

(b) Give the domain in interval notation.

$(-\infty, 2) \cup (2, \infty)$ 2 point

(c) Give the x-intercepts.

NONE 1 point



16. For the polynomial function $P(x) = x^4 - 2x^3 - 5x^2 + 4x + 6$, solve $P(x) = 0$.

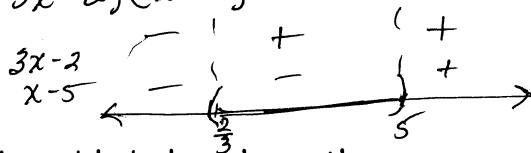
$-1 \mid 1 \ -2 \ -5 \ 4 \ 6$ $3 \mid 1 \ -3 \ -2 \ 4 \ 6$
 $\quad \quad -1 \ 3 \ 2 \ -6$ $\quad \quad 1 \ 0 \ -2 \ 0$
 $\hline 1 \ -3 \ -2 \ 6 \ 0$ $\hline 1 \ 0 \ -2 \ 0$

$x^2 = 2 \Rightarrow x = \pm\sqrt{2}$

16. $\{-1, 3, -\sqrt{2}, \sqrt{2}\}$ 4 points

17. Solve for x and write in interval form. $3x^2 < 17x - 10$

$3x^2 - 17x + 10 < 0$
 $(3x-2)(x-5) < 0$



17. $(\frac{2}{3}, 5)$ 3 points

18. The table below shows the average yearly income, in dollars, of individuals based on years of schooling. Use the calculator to model the data with a linear function.

Years of Schooling, x	Average Income, y
8	\$16,000
10	\$19,000
12	\$25,000
14	\$28,000

(a) $f(x) = 2.1x - 1.1$ in thousands 3 points

(b) Use the function to estimate the average yearly income for an individual with 16 years of schooling.

$f(16) = 32.5$

(b) \$32,500 2 points

19. Solve the equations for x:

(a) $4^{3x+1} = 16^{x+1}$

$$4^{3x+1} = 4^{2(x+1)}$$

$$3x+1 = 2x+2$$

$$x = 1$$

(a) 1 3 points

(b) $\log_2 x + \log_2(x-8) = 7$

$$\log_2 x(x-8) = 7$$

$$x^2 - 8x = 2^7$$

$$x^2 - 8x - 128 = 0$$

$$(x-16)(x+8) = 0$$

$x = 16$ or $x = -8$

(b) 16 3 points

20. If \$3500 is deposited in an account that pays 5.4% APR compounded continuously. How long will it take to double the original deposit?
(Round to the nearest tenth of a year) $P = P_0 e^{rt}$

$$7000 = 3500 e^{.054t}$$

$$2 = e^{.054t}$$

$$\ln 2 = .054t$$

$$\frac{\ln 2}{.054} = t \approx 12.8$$

20. 12.8 years 4 points

21. Graph the function. Show asymptotes and intercepts.

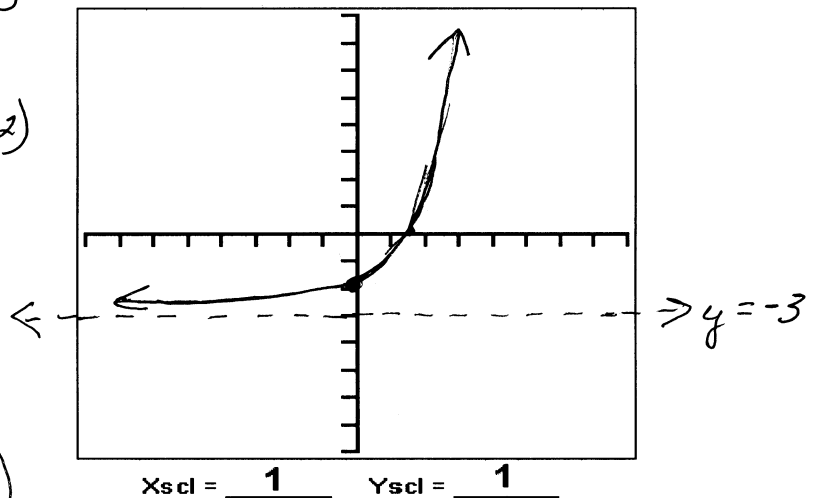
$F(x) = 2^x - 3$ h.a. $y = -3$

Let $x = 0$
 $y = 2^0 - 3 = 1 - 3 = -2$ (0, -2)

Let $y = 0$
 $0 = 2^x - 3$
 $3 = 2^x$
 $\ln 3 = x \ln 2$

$$\frac{\ln 3}{\ln 2} = x \approx 1.58 \quad (1.58, 0)$$

3 points



22. Solve for (x, y) using substitution or elimination: $-x + 2y = 2$
 $3x + y = 15$

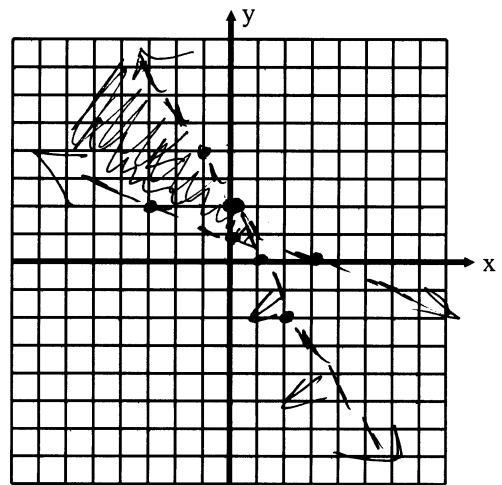
$$\begin{aligned} -3x + 6y &= 6 \\ 3x + y &= 15 \\ \hline 7y &= 21 \\ y &= 3 \\ -x + 2(3) &= 2 \\ -x + 6 &= 2 \\ -x &= -4 \\ x &= 4 \end{aligned}$$

22. $\{(4, 3)\}$
3 points

23. Graph the solution to the system of linear inequalities.

$$\begin{aligned} 2x + y &< 2 \\ x + 3y &> 3 \end{aligned}$$

$$\begin{aligned} y &< -2x + 2 \\ 3y &> -x + 3 \\ y &> -\frac{1}{3}x + 1 \end{aligned}$$



4 points

24. Solve for (x, y) algebraically: $x^2 + y^2 = 25$
 $2x + y = 10$

$$\begin{aligned} x^2 + (-2x + 10)^2 &= 25 \\ x^2 + 4x^2 - 40x + 100 &= 25 \\ 5x^2 - 40x + 75 &= 0 \\ x^2 - 8x + 15 &= 0 \\ (x - 3)(x - 5) &= 0 \\ x = 3 & \quad x = 5 \\ y = -6 + 10 = 4 & \quad y = 10 + 10 = 0 \\ (3, 4) & \quad (5, 0) \end{aligned}$$

24. $\{(3, 4), (5, 0)\}$
4 points

$$4x - y + z = -5$$

25. Consider the system: $2x + 2y + 3z = 10$

$$5x - 2y + 6z = 1$$

(a) Write a matrix equation equivalent to the system.

$$(a) \begin{bmatrix} 4 & -1 & 1 \\ 2 & 2 & 3 \\ 5 & -2 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -5 \\ 10 \\ 1 \end{bmatrix}$$

2 points

(b) Use your calculator and the inverse of the coefficient matrix of the matrix equation to solve the system.

$$(b) \{(-1, 3, 2)\}$$

2 points

26. Use a system of equations to solve the following:

A grocer sells oranges for \$0.95 each and grapefruit for \$1.05 each. You purchase a mix of 16 pieces of fruit and pay \$15.90. How many of each type of fruit did you buy?

$$x = \# \text{ oranges} \quad y = \# \text{ grapefruit}$$

$$m_{.95} \quad x + y = 16$$

$$m_{1.05} \quad .95x + 1.05y = 15.90$$

$$95x + 105y = 1590$$

$$-95x - 95y = -1520$$

$$10y = 70$$

$$y = 7$$

$$x + 7 = 16$$

$$x = 9 \quad \text{Notice}$$

$$26. \begin{array}{l} 9 \text{ oranges} \\ 7 \text{ grapefruit} \end{array}$$

4 points

A grade of "C" or better in Math 111 is required to take Math 115 or Math 215, or if this course is to be applied to a teaching degree.

A passing grade (D or better) is required to take Math 118 or for this course to satisfy the A₂, Mathematics component of the University Core Curriculum.