

Math 111
Final Exam

Name KEY
Section _____ Date _____

To receive full credit, show your work.

1. Find the equation of the line passing through the point (3, -2), and parallel to the line $y = -3x + 5$.

$$y = -3x + 5 \quad y + 2 = -3(x - 3)$$

$$m = -3 \quad y = -3x + 9 - 2$$

$$y = -3x + 7$$

- (a) $y = -3x + 7$ b. $y = -3x - 11$ c. $y = -3x + 11$ d. $y = -3x + 8$

1. a
2 points

2. $f(x) = 2x^2 - x + 5$. Find $f(x-1)$.

$$f(x-1) = 2(x-1)^2 - (x-1) + 5$$

$$= 2(x^2 - 2x + 1) - x + 1 + 5$$

$$= 2x^2 - 4x + 2 - x + 6 = 2x^2 - 5x + 8$$

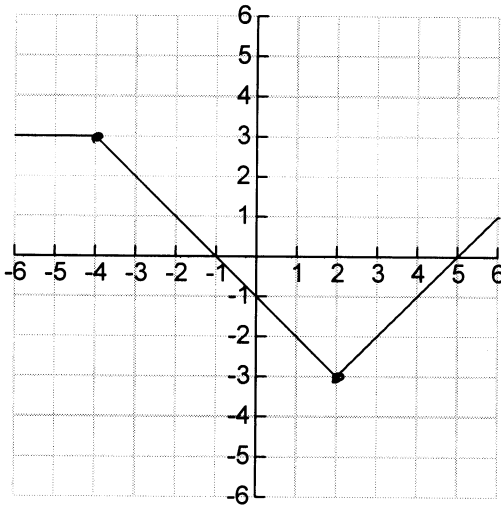
- a. $2x^2 - 5x + 7$ b. $2x^2 - x + 4$ c. $2x^2 - 5x + 6$ (d) $2x^2 - 5x + 8$

2. d
2 points

3. Refer to the graph of $f(x)$ below. Find $f(-4) - f(2)$

$$3 - (-3)$$

$$6$$



- a. 1 b. 2 c. 5 (d) 6

3. d
2 points

4. Use the quadratic formula to solve the equation, $x^2 + 2x = -10$. The solutions are:

$$a = 1 \quad b = 2 \quad c = 10$$

$$x = \frac{-2 \pm \sqrt{4 - 4(10)}}{2} = \frac{-2 \pm \sqrt{-36}}{2} = \frac{-2 \pm 6i}{2} = -1 \pm 3i$$

- a. $3+i, 3-i$ (b) $-1+3i, -1-3i$ c. $3+2i, 3-2i$ d. $1+3i, 1-3i$

4. b
2 points

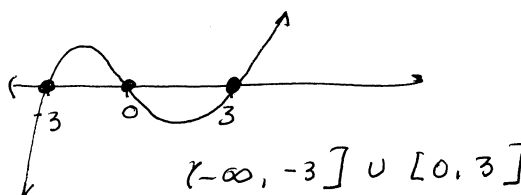
5. A rock falls from an airplane 576 ft. high. As it falls, its height, h , is given by the formula, $h = 576 - 16t^2$ where t represents time in seconds. When the rock hits the ground the height equals 0. How many seconds does it take to hit the ground?

$$\begin{aligned} 576 - 16t^2 &= 0 \\ -16t^2 &= -576 \\ t^2 &= 36 \\ t &= 6 \end{aligned}$$

- a. 4 b. 5 **c. 6** d. 7 5. c
2 points

6. Solve the inequality $x^3 - 9x \leq 0$. Express the answer in interval notation.

$$\begin{aligned} x^3 - 9x &= 0 \\ x(x^2 - 9) &= 0 \\ x(x-3)(x+3) &= 0 \\ x &= 0, \pm 3 \end{aligned}$$



- a. $[-3, 0] \cup [0, 3]$ **b. $(-\infty, -3] \cup [0, 3]$** c. $(-\infty, 0] \cup [-3, \infty)$ d. $[-3, 0] \cup [3, \infty)$
6. b
2 points

7. Solve the system of equations for x and y . $y = x^2 - 5x + 10$ The sum of x and y is:
 $3x - y = 6$

$$\begin{aligned} 3x - 6 &= x^2 - 5x + 10 \\ 0 &= x^2 - 8x + 16 \\ 0 &= (x-4)^2 \\ x &= 4 \quad y = 6 \end{aligned}$$

$$\begin{aligned} -y &= -3x + 6 \\ y &= 3x - 6 \end{aligned}$$

- a. 7 b. 8 c. 9 **d. 10** 7. d
2 points

8. The profit for a company's first 5 years of business is shown in the table. Use a graphing calculator to fit the data with a linear function, $y = ax + b$. Using the linear function, predict the profit in year 7.

Year (x)	1	2	3	4	5
Profit, in thousands, (y)	27	30	36	41	45

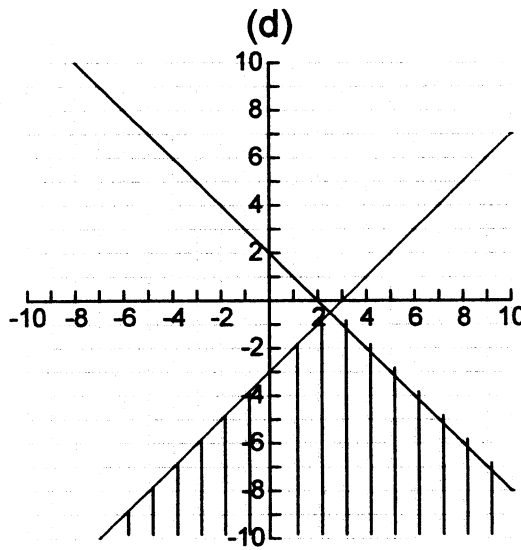
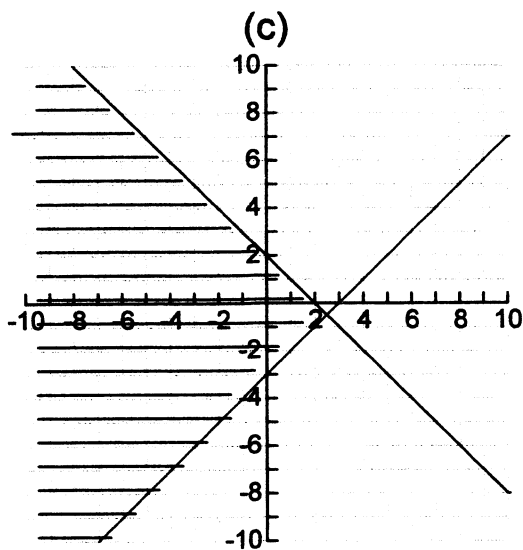
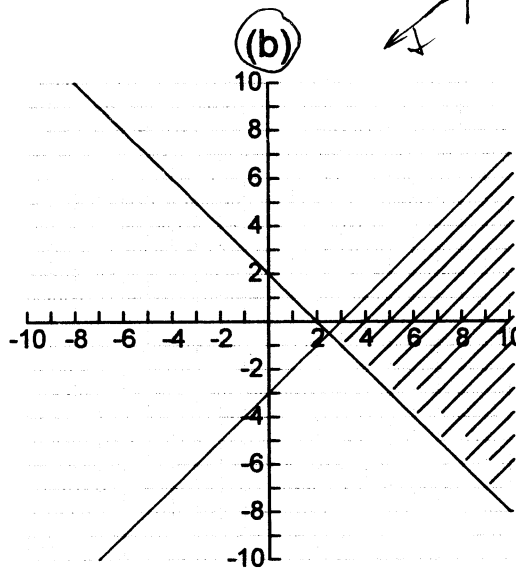
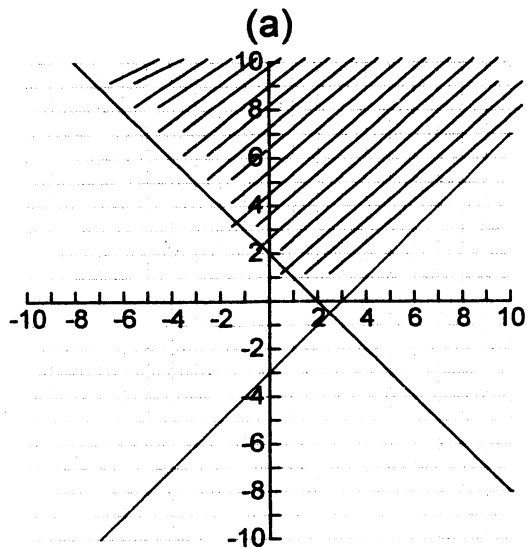
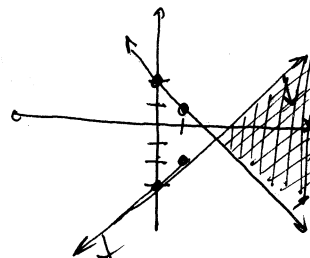
- a.** linear function profit, in thousands linear function profit, in thousands
 $y = 4.7x + 21.7$ 54.6 $y = 4.7x + 20.5$ 53.4
b. $y = 5.1x + 20.3$ 56.0 d. $y = 5.2x + 21.3$ 57.7

8. a
2 points

9. Graph the solution of the system of inequalities.

$$y \leq x - 3$$

$$y \geq -x + 2$$



9. b
2 points

10. The population of Kenya has an exponential growth rate of 2.1% and an initial population of 24 million people. Using the exponential growth function, $P(t) = P_0 e^{kt}$, find t , the number of years it takes the population to double. Round to the nearest whole year.

$$2 = e^{.021(t)}$$

$$\ln 2 = .021t$$

$$33 = t$$

a. 28 years b. 31 years **c. 33 years** d. 35 years

10. c
2 points

11. Factor Completely. $4n^3 + 108$

$$4(n^3 + 27)$$

$$4(n+3)(n^2 - 3n + 9)$$

11. $\frac{4(n+3)(n^2 - 3n + 9)}{2 \text{ points}}$

12. Write in simplest form with only positive exponents.

$$\left(\frac{x^{-4}y^3}{3x^{-5}y^8}\right)^3$$

$$\left(\frac{x}{3y^5}\right)^3 = \frac{x^3}{27y^{15}}$$

12. $\frac{x^3}{27y^{15}}$
3 points

13. Simplify.

$$\frac{x}{x^2 + 9x + 20} - \frac{4}{x^2 + 7x + 12}$$

$$\frac{x}{(x+4)(x+5)} - \frac{4}{(x+3)(x+4)}$$

$$\frac{x(x+3) - 4(x+5)}{(x+3)(x+4)(x+5)}$$

$$\frac{x^2 + 3x - 4x - 20}{LCD} = \frac{(x-5)(x+4)}{LCD} = \frac{x-5}{(x+5)(x+3)}$$

13. $\frac{x-5}{(x+5)(x+3)}$
3 points

14. Given two points A(4, -6) and B(1, 0).

a) Find the distance between A and B. Give exact value.

$$d = \sqrt{(4-1)^2 + (-6-0)^2}$$

$$= \sqrt{9 + 36} = \sqrt{45}$$

$$= 3\sqrt{5}$$

a. $\frac{d = 3\sqrt{5}}{2 \text{ points}}$

b) Find the slope and write the equation of the line AB in standard form.

$$m = \frac{-6}{4-1} = \frac{-6}{3} = -2$$

$$y - 0 = -2(x - 1)$$

$$y = -2x + 2$$

$$y = -2x + 2$$

b. $\frac{2x + y = 2}{3 \text{ points}}$

15. For $f(x) = x^2 + 10$ and $g(x) = \sqrt{x-1}$, find

a. $f \circ g = f(\sqrt{x-1}) = x - 1 + 10$
 $= x + 9$

a. $\frac{x + 9}{2 \text{ points}}$

b. $g(f(0)) = g(10) = \sqrt{9} = 3$

b. $\frac{3}{1 \text{ point}}$

16. Solve the equation $\frac{x}{x+4} = \frac{3}{x-1}$ $x \neq -4, 1$

$$x(x-1) = 3(x+4)$$

$$x^2 - x - 3x - 12 = 0$$

$$x^2 - 4x - 12 = 0$$

$$(x-6)(x+2) = 0$$

$$x = 6, -2$$

6. $\underline{\underline{\{6, -2\}}}$
3 points

17. Solve the inequality, writing your answer in interval notation.

$$-3 \leq 2(x+3) < 6$$

$$-3 \leq 2x+6 < 6$$

$$-9 \leq 2x \leq 0$$

$$-\frac{9}{2} \leq x \leq 0$$

7. $\underline{\underline{[-\frac{9}{2}, 0]}}$
3 points

18. Find the following for $f(x) = 2x^2 - x - 36$.

a. x-intercepts $2x^2 - x - 36 = 0$
 $(2x-9)(x+4) = 0$

a. $\underline{\underline{(\frac{9}{2}, 0), (-4, 0)}}$
2 points

b. y-intercept

b. $\underline{\underline{(0, -36)}}$
1 point

c. vertex $h = -\frac{b}{2a} = \frac{1}{4}$

$$k = f\left(\frac{1}{4}\right) = 2\left(\frac{1}{16}\right) - \frac{1}{4} - 36$$

$$= -\frac{289}{8} = -36\frac{1}{8}$$

c. $\underline{\underline{\left(\frac{1}{4}, -\frac{289}{8}\right)}}$
2 points

d. line of symmetry

d. $\underline{\underline{x = \frac{1}{4}}}$
1 point

19. Perform the following operations:

a. $(3-4i) + (4+2i)$

a. $\underline{\underline{7-2i}}$
1 point

b. $(3-4i) - (4+2i)$

b. $\underline{\underline{-1-6i}}$
1 point

c. $(3-4i)(4+2i) = 12 + 6i - 16i - 8i^2$
 $= 12 - 10i + 8$

c. $\underline{\underline{20-10i}}$
2 points

d. $\frac{(3-4i)(4-2i)}{(4+2i)(4-2i)} = \frac{12-6i-16i+8i^2}{16-4i^2}$
 $= \frac{4-22i}{20}$

d. $\underline{\underline{\frac{2}{10} - \frac{11}{10}i}}$
2 points

$$= \frac{2}{10} - \frac{11}{10}i$$

20. Consider the polynomial function $P(x) = (x+1)^3(x+2)(x-2)^2$.

a. Determine the degree of the polynomial.

a. $\frac{6^{th}}{1 \text{ point}}$

b. Determine the zeros of the function and state the multiplicity

3 points

ZERO VALUES

MULTIPLICITY

$x = -1$

$n = 3$

$x = -2$

$n = 1$

$x = 2$

$n = 2$

c. Determine the end behavior of the graph of the function.



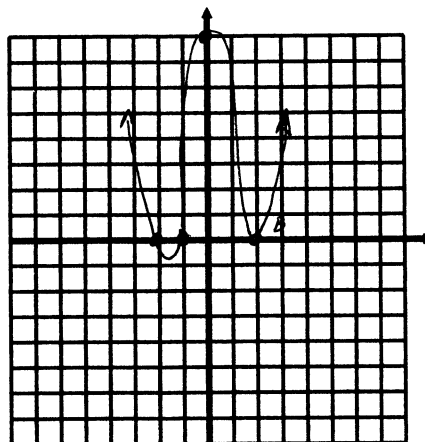
c. $\frac{\uparrow \uparrow}{2 \text{ points}}$

d. Determine the y-intercept of the function.

$P(0) = 2(-2)^2 = 8$

d. $\frac{(0, 8)}{1 \text{ point}}$

e. Sketch the graph of the function. 3 points



21. Given that 1 and 3 are zeros of the function $P(x) = x^4 - 4x^3 + 7x^2 - 16x + 12$, find the remaining zeros. Show all work to receive credit.

$$\begin{array}{r|rrrrr} 1 & 1 & -4 & 7 & -16 & 12 \\ & & 1 & -3 & 4 & 12 \\ \hline 3 & 1 & -3 & 4 & -12 & 12 \\ & & 3 & 0 & 12 & 12 \\ \hline & 1 & 0 & 4 & 0 & 0 \end{array}$$

$x^2 = -4$
 $x = \pm 2i$

21. $\frac{x = \pm 2i}{3 \text{ points}}$

22. Consider the function: $f(x) = \frac{(3x-1)(x+2)}{x^2+x-12}$
 $(x+4)(x-3)$

- a. Give the equation(s) of any vertical, horizontal, or oblique asymptotes of the function.

Vertical asymptotes: $x = -4, x = 3$
2 points

Horizontal asymptote: $y = 3$
1 point

Oblique asymptote: $none$
1 point

- b. Determine the coordinates of the x and y-intercepts

X-intercept(s): $(\frac{1}{3}, 0), (-2, 0)$
2 points

Y-intercept: $(0, \frac{1}{6})$
1 point

23. a. How would you obtain the graph of $f(x) = \log\left(x - \frac{5}{2}\right) - 1$ from the graph of $f(x) = \log x$?

horiz shift to the right $\frac{5}{2}$, vert shift down 1
2 points

- b. What is the domain of the function in part a? (Give the answer in set builder notation)

$x - \frac{5}{2} > 0$
 $x > \frac{5}{2}$

$\{x \mid x > \frac{5}{2}\}$
b. $(\frac{5}{2}, \infty)$
2 points

24. Suppose \$4500 is invested at 3.5%, compounded monthly. How much will be in the account after 3 years? $A = P\left(1 + \frac{r}{n}\right)^{nt}$

$P = 4500$
 $r = .035$
 $n = 12$
 $t = 3$

$A = 4500\left(1 + \frac{.035}{12}\right)^{12 \cdot 3}$

24. $\$4997.43$
3 points

25. Simplify using the rules of logarithms (must show steps and do not use calculator):

$$5\log_4 2 + \log_4 2$$

$$\begin{aligned} \log_4 2^5 + \log_4 2 \\ \log_4 2^{5 \cdot 2} &= \log_4 2^6 \\ &= \log_4 64 = 3 \end{aligned}$$

25. $\frac{3}{3 \text{ points}}$

26. Solve: $2\log_3 x - \log_3(x-2) = 2$

$$\log_3 x^2 - \log_3(x-2) = 2$$

$$\log_3\left(\frac{x^2}{x-2}\right) = 2$$

$$9 = \frac{x^2}{x-2}$$

$$x^2 = 9x - 18$$

$$\begin{aligned} x^2 - 9x + 18 &= 0 \\ (x-6)(x-3) &= 0 \\ x &= 6, 3 \end{aligned}$$

26. $\frac{x = 3, 6}{3 \text{ points}}$

27. Solve analytically: $2x + 3y = -1$
 $3x - y = 4$

$$-y = -3x + 4$$

$$y = 3x - 4$$

$$2x + 9x - 12 = -1$$

$$11x = 11$$

$$x = 1$$

$$\begin{aligned} 3 - y &= 4 \\ -y &= 1 \\ y &= -1 \end{aligned}$$

27. $\frac{(1, -1)}{3 \text{ points}}$

28. Write a system of equations and solve the following application. Toby split his savings into two different investments, one earning 5% and the other earning 7%. He put twice as much in the investment earning 7%. In one year he earned \$475 in interest. How much money did he invest in each account?

$$\begin{aligned} x &= \text{amt invested @ } 5\% \\ y &= \text{ " " " } 7\% \end{aligned}$$

$$2x = y$$

$$.05x + .07y = 475$$

$$.05x + .07(2x) = 475$$

$$.05x + .14x = 475$$

$$.19x = 475$$

$$x = 2500$$

$$y = 5000$$

28. $\frac{\$2500 @ 5\%}{\$5000 @ 7\%}$
4 points

29. Solve the system of equations using matrix methods. Write the matrix equation $AX = B$ and give solution in the form (x, y, z) .

$$2x - 2y + 3z = -1$$

$$2x - 6y - 4z = 9$$

$$x + y + z = -6$$

- a. Write the matrix equation, $AX = B$.

$$\begin{bmatrix} 2 & -2 & 3 \\ 2 & -6 & -4 \\ 1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -1 \\ 9 \\ -6 \end{bmatrix}$$

a. _____
2 points

- b. Find A^{-1} , fractions only.

$$A^{-1} = \begin{bmatrix} -\frac{1}{16} & \frac{5}{32} & \frac{13}{16} \\ -\frac{3}{16} & -\frac{1}{32} & \frac{7}{16} \\ \frac{1}{4} & -\frac{1}{8} & -\frac{1}{4} \end{bmatrix}$$

b. _____
2 points

- c. Give solution in ordered triple, fractions only.

$$\left(-\frac{109}{32}, \frac{-87}{32}, \frac{1}{8} \right)$$

c. _____
2 points

Notice

A grade of "C" or better in Math 111 is required to take Math 115 or Math 215, or if this course is to be applied to a teaching degree.

A passing grade (D or better) is required to take Math 118 or for this course to satisfy the A_2 , Mathematics component of the University Core Curriculum.