Math 111 Name _____
Final Exam Section _____ Date _____ Score Key

Please show all work for FULL CREDIT!

1. Given the following set of numbers: $\{-4.3\overline{9}, 15, -5, -\frac{3}{4}, \sqrt{11}, \sqrt[3]{-8}, 0, 4\frac{1}{3}, 0.838338333...\}$

a. Which number(s) are rational but NOT integers?

$-4.3\overline{9}, -\frac{3}{4}, 4\frac{1}{3}$

1 point

b. Which number(s) are irrational?

$\sqrt{11}$, $0.838338333...$

1 point

2. Simplify the expression:

$$\frac{3(6-9\div 3)-3\cdot 8}{(-2)^2+12^0}$$

$$= \frac{3(6-3)-24}{4+1} = \frac{3(3)-24}{5} = \frac{9-24}{5} = \frac{-15}{5} = -3$$

-3

3 points

3. Simplify: $\dfrac{\frac{x^2-y^2}{xy}}{\frac{x-y}{y}}$

$$= \frac{(x+y)(x-y)}{xy} \cdot \frac{y}{x-y}$$

$$= \frac{x+y}{x}$$

$\frac{x+y}{x}$

2 points

4. Simplify: $\left(3\sqrt{20}-4\sqrt{12}\right)\left(2\sqrt{5}-\sqrt{3}\right)$

$$= 6\cdot\sqrt{100} - 3\sqrt{60} - 8\sqrt{60} + 4\sqrt{36}$$

$$= 6\cdot 10 - 11\sqrt{60} + 4\cdot 6$$

$$= 60 - 11\cdot 2\sqrt{15} + 24$$

$$= 84 - 22\sqrt{15}$$

or $\left(3\sqrt{20}-4\sqrt{12}\right)\left(2\sqrt{5}-\sqrt{3}\right)$

$= (3\cdot 2\sqrt{5} - 4\cdot 2\sqrt{3})(2\sqrt{5}-\sqrt{3})$

$= (6\sqrt{5} - 8\sqrt{3})(2\sqrt{5}-\sqrt{3})$

$= 12\cdot 5 - 6\sqrt{15} - 16\sqrt{15} + 8\cdot 3$

$= 60 - 22\sqrt{15} + 24$

$= 84 - 22\sqrt{15}$ $84-22\sqrt{15}$

4 points

5. Find the slope-intercept equation of a line through the point (5,-2) that is perpendicular to the line $4x+3y=6$.

$3y=-4x+6$

$y=-\frac{4}{3}x+2$

$m_1=-\frac{4}{3}$

$m=\frac{3}{4}$

$y-y_1=m(x-x_1)$

$y-(-2)=\frac{3}{4}(x-5)$

$y+2=\frac{3}{4}x-\frac{15}{4}$

$y=\frac{3}{4}x-\frac{15}{4}-2$

$y=\frac{3}{4}x-\frac{23}{4}$

$y=\frac{3}{4}x-\frac{23}{4}$

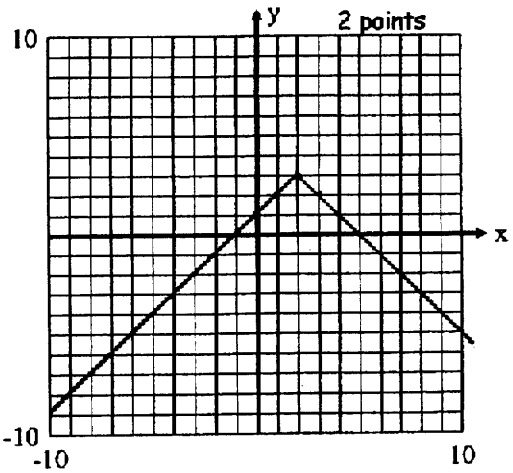
4 points

6. Fill in the appropriate blanks to describe how the graph of $g(x)=-|x-2|+3$ can be obtained from the graph of the graph of $f(x)=|x|$. **(1 point per blank)**

The graph of f(x) has been shifted horizontally ___right___ (left or right) exactly ___2___ (2 or 3) units, reflected over the ___x___ (x or y) axis, and finally, the graph has been shifted vertically ___up___ (up or down) exactly ___3___ (2 or 3) units.

Graph $g(x)$.

x	$g(x)=-\|x-2\|+3$
0	1
1	2
2	3
3	2
4	1



2 points

7. Let $f(x)=2x+5$ and $g(x)=\sqrt{16-x}$.

a) Find $(g)(-9)$. $=\sqrt{16-(-9)}=\sqrt{25}=5$

5

1 point

b) Find $(f\circ g)(7)=f(g(7))=f(\sqrt{16-7})=f(\sqrt{9})=f(3)$

$=2(3)+5$

$=11$

11

2 points

c) Find $(fg)(0)$.

$=f(0)\cdot g(0)$

$=5\cdot\sqrt{16}=5\cdot 4=20$

20

1 point

d) Find the domain of $f(x)$. Write it in interval notation.

$(-\infty,\infty)$

1 point

e) Find the domain of $g(x)$. Write it in interval notation.

$16-x\geq 0$

$x\leq 16$

$(-\infty,16]$

1 point

8. The table below shows the average tuition and fees at private colleges and universities in the given year. Let x represent the number of years after 1985.

Year, x	Average Tuition, y
1985, 0	5425
1990, 5	9431
1995, 10	12981
2000, 15	17171
2008, 23	26852

a) Use a calculator to fit a regression line to the data. Write the equation of the line.

$y = 915.5422446x + 4667.252207$

1 point

b) Use the equation to predict the average tuition, to the nearest dollar, in 2020.

36,711

2 points

c) State the correlation coefficient rounded to the thousandths place.

$r \approx 0.992$

1 point

9. Solve: $\left(\frac{4}{x+4} + \frac{9x}{x^2-16} = \frac{9}{x-4}\right)(x+4)(x-4)$

$LCD = (x+4)(x-4)$

$4(x-4) + 9x = 9(x+4)$

$4x - 16 + 9x = 9x + 36$

$4x = 52$

$x = 13$

{13}

3 points

10. Solve: $2x^2 - 5x + 4 = 0$

$a=2,\ b=-5,\ c=4$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$= \frac{-(-5) \pm \sqrt{(-5)^2 - 4(2)(4)}}{2(2)}$

$= \frac{5 \pm \sqrt{25-32}}{4}$

$= \frac{5 \pm \sqrt{-7}}{4}$

$= \frac{5 \pm \sqrt{7}i}{4}$

$= \frac{5}{4} \pm \frac{\sqrt{7}}{4}i$

$\frac{5}{4} \pm \frac{\sqrt{7}}{4}i$

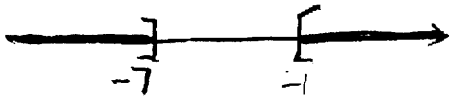
3 points

11. Solve the inequality: $|x+4| \geq 3$

Express your answer in interval notation.

$x+4 \geq 3$ or $x+4 \leq -3$

$x \geq -1$ $\quad$ $x \leq -7$



$(-\infty, -7] \cup [-1, \infty)$

4 points

12. Simplify: $(5-4i)(3+2i)$. Express in $a + bi$ form.

$= 15 + 10i - 12i - 8i^2$

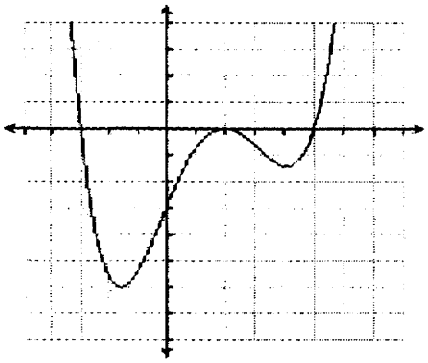
$= 15 - 2i + 8$

$= 23 - 2i$

$23 - 2i$

2 points

13. Answer the following based on the following graph.



a. The lowest possible degree of the function is 4

1 point

b. The leading coefficient of the function is **(positive or negative)** positive

1 point

c. The degree of the leading term is **(even or odd)** even

1 point

d. The constant term is **(positive or negative)** negative

1 point

e. The function has 3 distinct real zeros.

1 point

f. State the domain of f(x) in interval notation. $(-\infty, \infty)$

1 point

g. State the range of f(x) in interval notation. $[-6, \infty)$

1 point

14. Given that -2 is a zero of the function $f(x) = x^3 - 6x^2 - x + 30$, find the remaining zeros. **Show all work to receive credit.**

$$\begin{array}{r|rrr} -2 & 1 & -6 & -1 & 30 \\ & & -2 & 16 & -30 \\ \hline & 1 & -8 & 15 & 0 \end{array}$$

$x^2 - 8x + 15 = 0$

$(x-3)(x-5) = 0$

$x = 3$ or $x = 5$.

$\{3, 5\}$

4 points

15. Consider the function $f(x) = \dfrac{4x^2 - 11x - 3}{x^2 - 3x - 10} = \dfrac{(4x+1)(x-3)}{(x-5)(x+2)}$

Find all asymptotes and write your answer(s) in equation form. **If an asymptote does not exist state NONE.**

vertical asymptote(s) $x = 5,\ x = -2$ — 2 points

horizontal asymptote(s) $y = 4$ — 1 point

oblique asymptote (s) None — 1 point

Determine the coordinates of the x and y intercepts. **Write your answer as ordered pairs.**

x- intercept(s) $(-\frac{1}{4}, 0), (3, 0)$ — 2 points

y-intercept(s) $(0, \frac{3}{10})$ — 1 point

16. Solve the inequality. **Give the solution set in interval notation.**

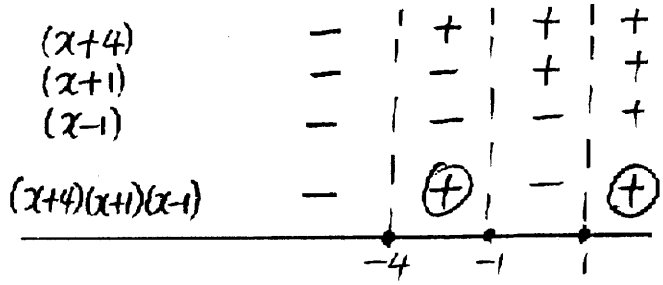
$x^3 + 4x^2 - x - 4 \geq 0$

$x^2(x+4) - (x+4) \geq 0$

$(x+4)(x^2-1) \geq 0$

$(x+4)(x+1)(x-1) \geq 0$

Critical values: $x = -4, -1, 1$



$[-4, -1] \cup [1, \infty)$

4 points

17. Solve for (x, y) algebraically: $\begin{cases} 6x+2y=-10 \\ y+6=-3x \end{cases}$ $\Rightarrow y=-3x-6$

$6x+2(-3x-6)=-10$

$6x-6x-12=-10$

$-12=-10$ False

ϕ

3 points

18. A model rocket is launched with an initial velocity of 90 ft/sec from a height of 50 ft. The height of the rocket t seconds after it has been thrown is given by the function

$$s(t)=-16t^2+90t+50$$

where t is the number of seconds after the rocket was launched.

a. Determine the time the rocket reaches its maximum height, find the height and round solutions to the nearest hundredth.

$-\frac{b}{2a}=-\frac{90}{2(-16)}=\frac{45}{16}\approx 2.81$

Time: 2.81 sec.

2 points

$S(\frac{45}{16})=176.56$

Height: 176.56 ft

2 points

b. How many seconds will it take for the rocket to return to the surface? Round solution to the nearest hundredth.

$S(t)=-16t^2+90t+50$

$0=-16t^2+90t+50$

Using the Zero feature. $t\approx 6.13$ sec

6.13 sec

2 points

19. Solve algebraically: $\begin{cases} 3x^2-y=0 \\ 5x-y=-2 \end{cases}$ $\Rightarrow y=3x^2$

$5x-3x^2=-2$

$0=3x^2-5x-2$

$0=(3x+1)(x-2)$

$3x+1=0$ or $x-2=0$

$\boxed{x=-\frac{1}{3}}$ $\boxed{x=2}$

If $x=-\frac{1}{3}$, $y=3(-\frac{1}{3})^2=\frac{1}{3}$

If $x=2$, $y=3(2)^2=3\cdot 4=12$

$(-\frac{1}{3},\frac{1}{3}),(2,12)$

4 points

20. Consider the system:
$$\begin{aligned} 2x - y + 4z &= -3 \\ x - 2y - 10z &= -6 \\ 3x \quad\quad + 4z &= 7 \end{aligned}$$

a) Write a matrix equation equivalent to the system:

$$\begin{bmatrix} 2 & -1 & 4 \\ 1 & -2 & -10 \\ 3 & 0 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -3 \\ -6 \\ 7 \end{bmatrix}$$

2 points

b) Use your calculator to solve the system. Write your answer as an ordered triple (x, y, z).

$\left(3, 7, -\frac{1}{2}\right)$

2 points

21. Solve the equation for x:

a) $3^{2x-2} = 27^{x+1}$

$3^{2x-2} = (3^3)^{x+1}$

$3^{2x-2} = 3^{3(x+1)}$

$3^{2x-2} = 3^{3x+3}$

$2x - 2 = 3x + 3$

$-5 = x$

$\{-5\}$

3 points

b) $\log_2(x+4) + \log_2(6-x) = 4$

$\log_2(x+4)(6-x) = 4$

$2^4 = (x+4)(6-x)$

$16 = 6x - x^2 + 24 - 4x$

$x^2 - 2x - 8 = 0$

$(x-4)(x+2) = 0$

$x = 4$ or $x = -2$

$\{4, -2\}$

3 points

c) $\ln x - \ln(x-3) = \ln 5$

$\ln \frac{x}{x-3} = \ln 5$

$\frac{x}{x-3} = 5$

$x = 5x - 15$

$-4x = -15$

$x = \frac{15}{4}$

$\left\{\frac{15}{4}\right\}$

3 points

22. A student has a total of 255 points on three tests. The sum of the scores on the second and third tests is 65 higher than the score on the first test. The second score exceeds the third by 14 points. Find the three scores using a system of equations.

let x = the score on the first test,
y = " " " 2nd test,
z = " " " 3rd test

$$\begin{cases} x+y+z=255 \\ y+z=x+65 \\ y=z+14 \end{cases} \Rightarrow \begin{cases} x+y+z=255 \\ -x+y+z=65 \\ y-z=14 \end{cases} \rightarrow \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix}=\begin{bmatrix} 255 \\ 65 \\ 14 \end{bmatrix}$$

(95, 87, 73)
4 points

23. Suppose $12,000 is invested at an interest rate of 5% APR compounded continuously.
How long will it take to double the original amount? (Round to the nearest tenth of a year) $P = P_0 e^{rt}$

$P(t) = P_0 e^{rt}$

$2(12{,}000) = 12{,}000\, e^{0.05t}$

$\ln 2 = \ln e^{0.05t}$

$\ln 2 = 0.05t$

$t = \frac{\ln 2}{0.05} \approx 13.9$

13.9 yr
4 points

Notice:

A grade of "C" or better in Math 111 is required to take Math 115 or Math 215, or if this course is to be applied to a teaching degree.

A passing grade of D or better is required to take Math 118 or for this course to satisfy the A_2 Mathematics component of the University Core curriculum.