Determining the level of output using unit isoquants

The level of \( Y^* \) is determined for given set of commodity prices and \( X^* \). This establishes \( w/r \) and the consequent capital labor ratios in the \( X \) and \( Y \) industries.

Letting \( E \) represent the endowment of labor and capital, we need to examine the distribution of these inputs in the production of \( X \) and \( Y \). Keeping in mind the assumption of full employment, then \( L_x + L_y = L_{total} \) and \( K_x + K_y = K_{total} \).

Summing up, we know we will be producing on \( k_x^* \) and \( k_y^* \) for the reasons given above and that the sums of the inputs employed in the two industries must be equal to the country’s endowment.

A vector in \( K, L \) space is defined by its slope an length. In the diagram above there are four vectors along the sides of a parallelogram where the vectors \( OA \) is the same as \( BE \), and \( OX \) is the same a \( AE \). The vectors that are the same are because they have the same slope and length. They are on opposites of the parallelogram. The vector \( OA \) running from the origin to point \( A \) describes amounts of inputs \( L_y, K_y \). The vector \( OB \) contains inputs of the amounts \( L_x, K_x \). If we sum the two they must add up to the total endowment. This is the case since \( OB \) is equal to \( AE \), and the sum of \( OA \) and \( AE \) equals the vector from the origin to \( E \), the endowment of labor and capital. Thus the amounts of \( X \) and \( Y \) produced given the endowment of \( L \) and \( K \), and given the initial set of commodity prices is described by the isoquants \( X@ \) and \( Y@ \).

Rybczynski Theorem - an increase in the supply of a factor or input will lead to an increase in the production of the good which uses the input intensively, while decrease the production of the other good.

\[ Y^* \]
\[ X^* \]
\[ k_y^* \]
\[ k_x^* \]
\[ L \]
\[ K \]
\[ E \]
\[ A \]
\[ B \]
\[ O \]
\[ L_y \]
\[ L_x \]
\[ K_y \]
\[ K_x \]
\[ X@ \]
\[ Y@ \]
To prove the theorem, we start with a clean version of the above diagram. Now we add labor to the country’s endowment. This is portrayed as a rightward shift of the endowment from point $E$ to $E'$. We recreate the parallelogram described above and determine the relative production levels of $X$ and $Y$ to the previous amounts of $X@$ and $Y@$. In this example an increase in the amount of labor has resulted in an increase in the production of good $X$ from $X@$ to $X^\wedge$. We know that $X$ in labor intensive by comparing capital labor ratios. The production of good $Y$, the capital intensive good fall from $Y@$ to $Y^\wedge$. Hence this example is consistent with the Rybczynski Theorem.