Math 111
Final Exam   Name______________________________

                      Section_________      Date__________________________

1. (a) Graph the solution set specified by the given statement. 2 points
   \[\{x \mid x > -3 \text{ and } x \leq 2\}\]

   \[\begin{array}{cccccccccccc}
   -10 & -9 & -8 & -7 & -6 & -5 & -4 & -3 & -2 & -1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
  \end{array}\]

   (b) Express the final solution set from part (a) in simplest interval format.

   Interval(s): ________________________

2. Write in simplest form with only positive exponents.
   \[\left(\frac{2}{2a^3b^{-4}}\right)^3 \left(-3a^\frac{1}{2}b^\frac{3}{2}\right)^2\]

2. __________________

4 points

3. Perform the indicated operations and simplify where possible. Show all work to receive credit.
   \[\frac{3y}{y^2 - 7y + 10} - \frac{2y}{y^2 - 8y + 15}\]

3. __________________

3 points

4. Write an equation for a function that has the shape of \(y = |x|\), but is shifted left 4 units and up 3 units.

4. __________________

2 points
5. The points (-2, 3) and (4, -5) are the endpoints of the diameter of a circle. Find the length of the radius of the circle.

6. Find the equations of the line that passes through the point (-1, 5) and is perpendicular to the line $3x - 4y = -8$. Write in slope-intercept form.

7. A graph of $y = f(x)$ is given below. No formula for $f$ is given.
   
   Graph $y = -\frac{1}{3} f(x)$.

8. Solve the inequality $|y + 5| \geq 2$ giving the solution in interval notation.
9. Solve for $a$: \[ \frac{5}{a-4} - \frac{3}{a-1} = \frac{a+1}{a-4} \]

10. Consider the function $g(x) = x^2 - 15x + 36$ and find:
   a) the zeros of $g(x)$
   b) the vertex of the graph of $g(x)$
   c) the range of $g(x)$ (in interval notation)

11. Solve for $b$: $-2(b+3)^2 = 40$
12. A ball is thrown vertically upward with an initial speed of 48 ft/s. Its height, in feet, after t seconds is given by \( h(t) = -16t^2 + 48t \). Find the maximum height of the ball and the time when it reaches that height.

13. Use division to find the quotient \( Q(x) \) and the remainder \( R(x) \), and express \( P(x) \) in the form \( d(x) \cdot Q(x) + R(x) \).

\[
P(x) = x^3 - 2x^2 + x - 6
\]
\[
d(x) = x + 3
\]

14. Find a polynomial of degree 4 with -2 as a zero of multiplicity 2 and 0 and 3 as zeros of multiplicity 1 and write as \( P(x) = a_n x^n + a_{n-1} x^{n-1} + a_{n-2} x^{n-2} \ldots + a_1 x + a_0 \).
15. Make a graph of \( f(x) = \frac{6}{(x-2)^2} \).

(a) Label all the asymptotes and y-intercept. \(3 \text{ points}\)

(b) Give the domain in interval notation. \(2 \text{ points}\)

(c) Give the x-intercepts. \(1 \text{ point}\)

16. For the polynomial function \( P(x) = x^4 - 2x^3 - 5x^2 + 4x + 6 \), solve \( P(x) = 0 \).

16. \(4 \text{ points}\)

17. Solve for \( x \) and write in interval form. \( 3x^2 < 17x - 10 \)

17. \(3 \text{ points}\)

18. The table below shows the average yearly income, in dollars, of individuals based on years of schooling. Use the calculator to model the data with a linear function.

<table>
<thead>
<tr>
<th>Years of Schooling, ( x )</th>
<th>Average Income, ( y )</th>
</tr>
</thead>
<tbody>
<tr>
<td>8</td>
<td>$16,000</td>
</tr>
<tr>
<td>10</td>
<td>$19,000</td>
</tr>
<tr>
<td>12</td>
<td>$25,000</td>
</tr>
<tr>
<td>14</td>
<td>$28,000</td>
</tr>
</tbody>
</table>

(a) \(3 \text{ points}\)

(b) Use the function to estimate the average yearly income for an individual with 16 years of schooling.

(b) \(2 \text{ points}\)
19. Solve the equations for $x$:

(a) $4^{3x+1} = 16^{x+1}$  

(b) $\log_2 x + \log_2 (x-8) = 7$

20. If $3500$ is deposited in an account that pays 5.4% APR compounded continuously. How long will it take to double the original deposit? (Round to the nearest tenth of a year)  
$P = P_0 e^{rt}$

21. Graph the function. Show asymptotes and intercepts.  
$F(x) = 2^x - 3$
22. Solve for \((x, y)\) using substitution or elimination:
\[-x + 2y = 2\]
\[3x + y = 15\]

23. Graph the solution to the system of linear inequalities.
\[2x + y < 2\]
\[x + 3y > 3\]

24. Solve for \((x, y)\) algebraically:
\[x^2 + y^2 = 25\]
\[2x + y = 10\]
25. Consider the system: 
\[ 4x - y + z = -5 \]
\[ 2x + 2y + 3z = 10 \]
\[ 5x - 2y + 6z = 1 \]

(a) Write a matrix equation equivalent to the system.

(b) Use your calculator and the inverse of the coefficient matrix of the matrix equation to solve the system.

26. Use a system of equations to solve the following:

A grocer sells oranges for $0.95 each and grapefruit for $1.05 each. You purchase a mix of 16 pieces of fruit and pay $15.90. How many of each type of fruit did you buy?

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Notice

A grade of "C" or better in Math 111 is required to take Math 115 or Math 215, or if this course is to be applied to a teaching degree.

A passing grade (D or better) is required to take Math 118 or for this course to satisfy the A2, Mathematics component of the University Core Curriculum.