Tuition Hikes and Student Performance

Sudesh Mujumdar, University of Southern Indiana
Timothy Schibik, University of Southern Indiana
Daniel Friesner, Gonzaga University
Mohammed Khayum, University of Southern Indiana

ABSTRACT
This paper develops a simple time-allocation model to examine the relationship between college tuition and students' performance at college studies. We find that an increase in tuition lowers performance. Further, it is found that an increase in the wage rate lowers performance (since it results in an increase in the hours spent working). Our model thus (indirectly) provides theoretical support to the empirical finding that an increase in the minimum wage may actually increase employment, established through the studies of (among others) Card (1992a, 1992b), and Card and Krueger (1994). We also examine how the mean and variance of the students' household income affects the average performance level. We find that an increase in the mean raises performance. Interestingly, a change in the variance that leaves the mean unchanged has no effect on the average performance level.

INTRODUCTION
Annual increases in tuition have become a staple feature of the American college-education picture (see Kline and Murray-Plumer, 1998; Levinson, 2001). While most parents do contribute (in differing degrees) towards meeting the college expenses of their children, their levels of support have not been rising in proportion to the tuition increases (see Kline and Murray-Plumer, 1998). The onus, then, has fallen on the student to come up with larger and larger sums of money to close the “funding-gap.” Students have sought to meet this burden through increased borrowing and working additional hours at their part-time jobs (or taking on employment if they were not employed). The increased “work-responsibility” has, expectedly, not had a salutary effect (to say the least) on their performance at college studies (see Wehrman, 2003).

The above discussion, then, suggests a negative relationship between changes in tuition amounts and changes in performance at college studies. The primary objective of the present paper is to make a first attempt at formally investigating this “tuition-performance” relationship. One might wonder as to the purpose of a formal investigation since the underlying logic of this relationship seems quite clear. A formal investigation, however, may help shed light on questions where pure intuition offers little guidance or the guidance that is offered lacks a firm footing. Some such questions are:

a) Would an increase in the “teenage-wage” cause students to work less and spend more time on their studies, resulting in better grades?

b) Suppose at a certain college, the relatively rich parents become richer and the relatively poor parents become poorer (implying that the variance of the

---

2 This negative effect has been more pronounced for low-income students (see Wehrman, 2003).
3 One might argue, here, that even if a student works longer hours in response to a tuition hike, her performance at college studies may not suffer if she lowers her course-load. It is true that some students do, in fact, lower their course-load, but a large majority of the students seek to maintain their “usual” course-loads (see Wehrman, 2003).
4 The investigation will be largely exploratory in nature at this stage of development of the paper. More depth will be achieved as our work progresses.
college’s household income increases), how will this affect the average performance of the students at their studies?

Further, our theoretical work can provide the foundation for conducting a “rigorous” empirical study of the “tuition-performance” relationship in the sense that it can help better specify the econometric model(s) that will be employed in the inquiry.

The rest of the paper is organized as follows. The next section develops a very simple model (in a somewhat general form) regarding students’ time-allocation decisions. In Section 3, specific functional forms are employed in the setup of the model (developed in Section 2) to facilitate the study of the “tuition-performance” relationship. This version of the model is also well suited for addressing the above questions: (a) and (b). Section 4 undertakes this “addressing” task. Section 5 offers some remarks on our future course of action with regard to the development of this paper.

THE GENERAL MODEL

From the discussion in the previous section, it appears that a time allocation model (a la Becker, 1965) presents a suitable tool for a more careful exploration of the “tuition-performance” relationship. Using the Becker-framework, we will build a very simple model that seeks to capture a student’s decision on how to allocate her time between work and study – the two activities that are relevant to our concern.\(^5\)

Suppose a student has to decide at, say, the beginning of a semester on how to allocate her endowment of time \((T)\) between work and study (for the semester). Now, (in the Becker-\(\text{vein}\)) the student spends time studying because she values, not the activity itself, but its “immediate” outcome – represented by, say, the numerical scores obtained on courses (e.g., a composite score of 95% in Physics 101, 90% in Calculus I, etc) or the G.P.A for the semester. She works at a part-time job to finance some of her college-living expenses. The expenses may be incurred for things such as room-rent, “eating-out”, “ordering-in,” renting movies, etc. Let us lump these “expense-items” into one and dub it the Composite good \((C)\).\(^6\) Some parental support is received towards financing her college-living expenses. For simplicity, we assume that tuition is paid through parental support and/or financial aid. This assumption allows us to focus on only one dimension of the “financial crunch” effect that students face under the above described phenomenon of parental support not keeping pace with tuition increases. We can maintain that if tuition goes up, parents foot the increase but lower their contribution towards living expenses – thus the “financial crunch” effect is experienced only in the living-expenses dimension (of financing a college education). We are now in a position to formally set up the student’s problem.

Suppose there are \(N\) students in a college. Student \(i\)’s utility function is given by:

\[
U_i = U_i(P, C_i)
\]

where \(P\) is just some (numerical) measure of performance at college studies. Further, \(\frac{\partial U_i}{\partial P} > 0, \frac{\partial^2 U_i}{\partial P^2} < 0, \frac{\partial U_i}{\partial C} > 0, \frac{\partial^2 U_i}{\partial C^2} < 0\). That is, utility increases but at a diminishing rate in both arguments – this is the standard diminishing marginal utility assumption.\(^7\)

\(^5\) One might argue, here, that there are other activities - “leisure”, sleep - that, although not directly relevant to the authors’ concern, are perhaps as important to the student as the activities considered and hence must be accorded their due. Explicitly accounting for these other activities may very well yield additional insights but will not, we maintain, change the main thrust of our conclusions.

\(^6\) While time is allocated towards “acquisition” of the Composite good, no time is allocated towards consumption of the Composite good. This is a simplifying assumption that is standard in (“consumer”) time-allocation models.

\(^7\) The reason for the subscript \(i\) on \(C\) in the utility function will soon become apparent.
Let the following function describe the “technology” by which time spent on study \( (t_p) \) produces performance:

\[
P = P(t_p), \quad \frac{\partial P}{\partial t_p} > 0, \quad \frac{\partial^2 P}{\partial t_p^2} < 0.
\]

These derivatives indicate that the greater the amount of time spent on studies the greater the level of performance, but the gain to the level of performance from each additional unit of time spent on studies decreases.

To finance consumption of \( C_i \), student \( i \) has two sources; her part-time job and parental support. Let \( t_c \) denote the time spent on work. Let \( w \) denote the wage rate (for some given unit of time - \( t_c \) and \( t_p \) are, then, measured in that unit of time). The parental support received by student \( i \) \((M_i)\) is given by:

\[
M_i = \beta \frac{H_i}{T_u}, \quad \text{where} \quad \beta > 0, \quad T_u \text{ is the tuition amount for a semester at a certain college for the “usual” course-load and } H_i \text{ is the combined income of } i \text{'s parents – let us refer to this amount as the household income of student } i. \text{ Now, note that for a given tuition amount, the higher the household income, the higher is the level of parental support. Further, for a given level of household income, if tuition increases, the level of parental support drops. Thus student } i \text{'s “consumption constraint” is given by:}
\]

\[
C_i = t_c w + \beta \frac{H_i}{T_u}
\]

From the above discussion, it follows that the students’ utility functions are identical with respect to “performance” but not with respect to consumption of the Composite good. A certain amount of time spent on study yields an identical level of performance (across students) and hence makes the same contribution to utility irrespective of the student under consideration (for a given level of consumption of the Composite good). However, given some level of “performance,” a certain amount of time spent on work yields different levels of consumption (if household incomes vary) and makes different contributions to students’ utilities.

We are now in a position to state student \( i \)’s problem. The objective of student \( i \) is to

\[
\text{Max } U_i = U_i(P(t_p), C_i) \text{ with respect to } t_c
\]

subject to

(i) \( C_i = t_c w + \beta \frac{H_i}{T_u} \) \((\text{the “consumption” constraint})\)

and

(ii) \( T = t_c + t_p \) \((\text{the “time” constraint})\)

This problem can be re-written as

\[
\text{Max } U_i = U_i(P(T - t_c), t_c w + \beta \frac{H_i}{T_u}) \text{ with respect to } t_c
\]

The first order condition (F.O.C) is given by

\[
- \frac{\partial U_i}{\partial P} \frac{\partial P}{\partial t_p} + \frac{\partial U_i}{\partial C_i} w = 0
\]

Simplifying (5) yields

\[
\frac{\partial U_i}{\partial C_i} \frac{\partial C_i}{\partial U_i} = \frac{\partial P}{\partial t_p} \frac{\partial P}{w}
\]

Equation (6) states that student \( i \) should allocate her time between work and study in such a way that the marginal rate of substitution of “consumption” for “performance” equals the ratio of the “return” to the given unit of time.
spent on study, to the “return” for the same (given) unit of time spent on work. (This is akin to the familiar condition for optimum allocation of income between the consumption of two goods; good 1 and good 2 - MRS of good 2 for good 1 = ratio of the price of good 1 to the price of good 2).

Now, what we wish to determine is how \( t_c \) and consequently \( t_p \) vary with \( T_u \). This can be accomplished by totally differentiating (6) with respect to \( T_u \) and then solving for \( \frac{\partial t_c}{\partial T_u} \).

Doing so yields an expression whose sign is cannot be determined unless we impose additional “structure” on the utility function and whose component terms cannot easily be interpreted to say anything of substance about \( \frac{\partial t_c}{\partial T_u} \). To overcome this we will, in the following section, work with specific functions. Further the specific-functions-form of the model makes it relatively easy for us to conduct our desired experiments.

THE “SPECIFIC” FORM OF THE MODEL

We assume that the utility function takes on the following (“Cobb-Douglas”) form:

\[
U_i = P^\alpha C_i^{1-\alpha}, \text{ where } 0 < \alpha < 1
\]

Further, we assume that the “performance function” takes on the following form:

\[
P = (t_p)^\theta, \text{ where } 0 < \theta < 1
\]

Student \( i \)’s problem can now be stated as:

\[
\max U_i = [(t_p)^\theta]^\alpha C_i^{1-\alpha} \quad \text{with respect to } t_c
\]

subject to

\[
\frac{C_i}{T} = t_c w + \beta \frac{H_i}{T_u} \quad \text{…(the “consumption” constraint)}
\]

and

\[
T = t_c + t_p \quad \text{(the “time” constraint)}
\]

The above problem can be more simply stated as:

\[
\max U_i = [(T - t_c)^\theta]^{\alpha} \left[ t_c w + \beta \frac{H_i}{T_u} \right]^{1-\alpha} \quad \text{with respect to } t_c
\]

The F.O.C. is given by

\[
\frac{\partial U_i}{\partial t_c} = [(T - t_c)^\theta]^{\alpha} \left[ t_c w + \beta \frac{H_i}{T_u} \right]^{1-\alpha} - [\alpha \theta (T - t_c)^{\theta - 1} + \theta (T - t_c)^\theta] w = 0
\]

Solving (9) for \( t_c \) yields

\[
t_c = \frac{T(1-\alpha)w - \beta \frac{H_i}{T_u}}{\alpha \theta \left[ w + w(1-\alpha) \right]}
\]

Note that an increase in tuition (ceteris paribus) results in an increase in the time spent working (and, consequently, a decrease in the time spent on “studies”). This leads us to the following result.

**Proposition 1:** An increase in tuition results in a fall in a student’s performance at college studies.

Also, note that for a given a level of tuition, the higher the household income, the less is the time that is spent working and, consequently, the
greater is time spent on study resulting in better “performance.” There is some empirical evidence to support the finding that (on average) higher household income results in “better” performance (see e.g., Lee and Barro, 2001)

Lemma 1: An increase in \( w \) raises \( t_c \).

Proof: (10) can be rewritten as

\[
(11) \quad t_c = \frac{T(1-\alpha)w - \beta H_i}{\alpha \theta + (1-\alpha)} - \frac{\beta H_i}{T_w} \left[ 1 + \frac{(1-\alpha)}{\alpha \theta} \right]
\]

Now, (11) can be rewritten as

\[
(12) \quad t_c = \frac{T(1-\alpha)w}{\alpha \theta + (1-\alpha)} - \frac{\beta H_i}{T_w} \left[ 1 + \frac{(1-\alpha)}{\alpha \theta} \right]
\]

An increase in \( w \) lowers the value of the second expression on the “Right-Hand-Side” (R.H.S) of (12), resulting in an increase in \( t_c \). Q.E.D.

The finding embodied in the above lemma – that an increase in the teenage wage results in an increase in the hours (if an hour is our time unit) spent working – may appear somewhat counterintuitive; when an increase in the wage rate makes it possible for a student to meet her living expenses by working fewer hours, why does she choose to work more hours? The answer is found in the fact that the opportunity cost of each hour not spent working is now higher. Further, Lemma 1 lends theoretical support to the now well known empirical finding that moderate increases in the minimum wage may, in fact, increase “teenage” employment (and employment in general), established through the studies of (among others) Card (1992a, 1992b), and Card and Krueger (1994). Now, Bhaskar and To (1999) also provide theoretical support for this finding through a model of monopsonistic competition.

Here an increase in the wage rate increases the participation rate of the “high reservation wage workers” leading to an increase in employment (at the firm level). Thus, while the nature of our model is different from that of Bhaskar and To’s, the underlying rationale for why an increase in the wage generates more employment in both models is the same; the opportunity cost of an hour not spent on work goes up. The contribution of our model, then, on this issue of providing theoretical support for the mentioned empirical finding is to offer another (theoretical) leg on which to rest the finding.

Proposition 2: An increase in the “teenage wage” lowers a student’s performance at college studies.

Proof: follows from Lemma 1

Proposition 3: An increase in the supply of less skilled workers improves a student’s performance at college studies.

Proof: If less skilled workers are viewed as a substitute for “college” workers, then an increase in the supply of less skilled workers will lower the “teenage” wage or the “less skilled” wage. This will result in a student spending fewer hours at work and more hours at study leading to better performance at college studies. Q.E.D.

SOME AGGREGATE-LEVEL ANALYSES

We have already seen that an increase in a student’s household income results in a higher level of performance at college studies. It is common in casual conversation to make the leap from this observation to maintaining that an increase in a college’s average household income will lead to an increase in the average level of performance of the students. Is this leap legitimate? Again, pure intuition does not offer clear guidance on this question. The
The level of performance of student $i$, $(P_i)$, can thus be written as follows:

$$P_i = \left[ T - \frac{T(1-\alpha)}{\alpha \theta + (1-\alpha)} + \frac{\beta H_i}{T_p w[\alpha \theta + (1-\alpha)]} \right]^\theta$$

One can rewrite the above expression as:

$$(P_i)^{1/\theta} = \left[ \frac{\alpha \theta T}{\alpha \theta + (1-\alpha)} + \frac{\beta H_i \alpha \theta}{T_p w[\alpha \theta + (1-\alpha)]} \right].$$

Summing $(P_i)^{1/\theta}$ over our $N$ students yields:

$$\sum_{i=1}^{N} (P_i)^{1/\theta} = N \frac{\alpha \theta T}{\alpha \theta + (1-\alpha)}$$

$$+ \frac{\beta \alpha \theta}{T_p w[\alpha \theta + (1-\alpha)]} \sum_{i=1}^{N} H_i$$

Dividing both sides of this equality by $N$ yields:

$$\frac{\sum_{i=1}^{N} (P_i)^{1/\theta}}{N} = \frac{\alpha \theta T}{\alpha \theta + (1-\alpha)}$$

$$+ \frac{\beta \alpha \theta}{T_p w[\alpha \theta + (1-\alpha)]} \frac{\sum_{i=1}^{N} H_i}{N}$$

where $\bar{H} = \frac{\sum_{i=1}^{N} H_i}{N}$ is the average (or mean) household income of the students. Now, note that an increase in $\bar{H}$ produces an increase in $\frac{\sum_{i=1}^{N} (P_i)^{1/\theta}}{N}$. It is, of course, easily seen that if $\frac{\sum_{i=1}^{N} P_i}{N}$ increases then it must be that $\frac{\sum_{i=1}^{N} P_i}{N}$ (which is the average performance level of the students) also increases. Q.E.D.

Now, how would the average level of performance be affected if, say,

(i) the “high-income” households experienced an increase in their income levels and the “low-income” households experienced a decrease in their income levels, or

(ii) just the “high-income” households experienced an increase in their income levels, or

(iii) just the “low-income” households experienced an increase in their income levels.

More generally, how would the average performance level of the students change with a change in the variance of household income ($H$)? The following propositions address this question.
Proposition 5: A change in the variance of household income that leaves the mean household income unchanged will have no effect on the average performance level of the students.

Proof: Consider the expression:

\[ \frac{\sum_{i=1}^{N} (P_i)^{1/\theta}}{N} = \frac{\alpha \theta T}{\alpha \theta + (1 - \alpha)} + \frac{\beta \alpha \theta}{T_n \{\alpha \theta + (1 - \alpha)\} H} \]

This can also be written as:

\[ \frac{\sum_{i=1}^{N} (P_i)^{1/\theta}}{N} = \frac{\alpha \theta T}{\alpha \theta + (1 - \alpha)} + \frac{\beta \alpha \theta}{T_n \{\alpha \theta + (1 - \alpha)\} \left[ \frac{\sum_{i=1}^{N} H_i^2}{N} - \text{var}(H) \right]} \]

where

\[ \text{var}(H) = \frac{\sum_{i=1}^{N} H_i^2}{N} - \left( \bar{H} \right)^2. \]

First, note that \( \frac{\sum_{i=1}^{N} H_i^2}{N} > \text{var}(H) \), since \( \text{var}(H) \geq 0 \). A change in the \( \text{var}(H) \) that leaves \( \bar{H} \) unchanged must produce the same change in \( \frac{\sum_{i=1}^{N} H_i^2}{N} \) (this follows from the definition of the \( \text{var}(H) \)). Hence, \( \frac{\sum_{i=1}^{N} (P_i)^{1/\theta}}{N} \) will remain unchanged implying that \( \frac{\sum_{i=1}^{N} P_i}{N} \) has not changed. Q.E.D.

Let us give an idea of the intuition behind the above result. Suppose there is an increase in the variance of household income that comes about as in (i) described above, with the mean household income remaining unchanged. Here, the increase in the performance levels of the students whose household incomes have gone up (in total) exactly “makes up” for the decrease in the performance levels of students whose household incomes have gone down (in total). Hence, the average performance level of the students experiences no change.

Proposition 6: An increase in the variance of household income that increases the mean household income will increase the average performance level of the students.

Proof: Consider the expression:

\[ \frac{\sum_{i=1}^{N} (P_i)^{1/\theta}}{N} = \frac{\alpha \theta T}{\alpha \theta + (1 - \alpha)} + \frac{\beta \alpha \theta}{T_n \{\alpha \theta + (1 - \alpha)\} \left[ \frac{\sum_{i=1}^{N} H_i^2}{N} - \text{var}(H) \right]} \]

An increase in the \( \text{var}(H) \) that increases \( \bar{H} \) implies that the value of

\[ \left[ \frac{\sum_{i=1}^{N} H_i^2}{N} - \text{var}(H) \right] \]

increases (this follows from the definition of the \( \text{var}(H) \)). An increase in the value of

\[ \left[ \frac{\sum_{i=1}^{N} H_i^2}{N} - \text{var}(H) \right] \]

means that
\[
\sum_{i=1}^{N} (P_i)^{1/\theta} \frac{1}{N} \text{ is higher, which must imply that } \\
\sum_{i=1}^{N} \frac{P_i}{N} \text{ is higher. Q.E.D.}
\]

**Proposition 7:** A decrease in the variance of household income that decreases the mean household income will decrease the average performance level of the students.

**Proof:** The proof is similar to that of Proposition 6.

**FURTHER DEVELOPMENT OF THE PAPER**

The time-allocation model developed in this paper is quite “restrictive.” Some restrictions run as follows:

(i) It does not make room for the possibility that a student may choose to take on more debt when the level of parental support drops.

(ii) For a given level of household income, a certain amount of time that is spent on study yields the same level of performance which, in turn, makes the same contribution to utility for each student.

(iii) A student makes the time-allocation decision at the beginning of the semester and is not allowed to make another time-allocation decision during the course of the semester.

In our future work on this paper, we plan to do away or ease these restrictions. Further, we plan on collecting “student data” from, say, a particular college (to start with) to test the predictions of this paper.

**REFERENCES**


