Quiz III - 8am

$$I = \int \frac{dt}{4t^2 + 9}$$

This is reminiscent of the standard form

$$(*) \ldots \quad \int \frac{dx}{1+x^2} = \tan^{-1} x + C$$

Then we want to put it in a form like $(*)$:

$$I = \int \frac{dt}{9\left(\frac{4}{9}t^2 + 1\right)}$$

Let $u = \frac{2t}{3}$

so $du = \frac{2}{3}dt$

OR $dt = \frac{3}{2}du$

$$= \frac{1}{9} \int \frac{\frac{3}{2}du}{u^2+1}$$

$$= \frac{1}{9} \cdot \frac{3}{2} \int \frac{du}{u^2+1}$$

This is exactly the form $(*)$

$$= \frac{1}{6} \tan^{-1} u + C \quad \text{using } (*)$$

$$= \frac{1}{6} \tan^{-1}\left(\frac{2t}{3}\right) + C$$

Quiz III – 1pm

$$I = \int \frac{dt}{\sqrt{9-4t^2}}$$

This is reminiscent of the integral

$$(*) \ldots \quad I' = \int \frac{dx}{\sqrt{1-x^2}} = \sin^{-1}x + C$$

We want to put this in the <u>exact</u> form $(*)$:

$$I = \int \frac{dt}{\sqrt{9\left(1-\frac{4t^2}{9}\right)}} = \int \frac{dt}{\sqrt{9}\sqrt{1-\left(\frac{2t}{3}\right)^2}}$$

$$= \frac{1}{3}\int \frac{\frac{3}{2}\,du}{\sqrt{1-u^2}}$$

Let $u = \frac{2t}{3}$, so

$du = \frac{2}{3}\,dt$

~~or~~ $dt = \frac{3}{2}\,du$

$$= \frac{1}{2}\int \frac{du}{\sqrt{1-u^2}}$$

… which is <u>exactly</u> the form we want

$$= \frac{1}{2} \sin^{-1}u + C \quad \text{using } (*)$$

$$= \frac{1}{2}\sin^{-1}\left(\frac{2t}{3}\right) + C$$