Math 111 Final Exam Form A

Section _____ ***Show all work or document calculator usage to receive full credit.***

1. Determine whether the given pair of lines is parallel, perpendicular or neither. Show all work and give a reason for your answer.

$\begin{cases} y = 4x - 5 \\ 4y = 8 - x \end{cases}$

$y = 4x - 5 \Rightarrow m_1 = 4$ ✓

$4y = 8 - x \Rightarrow \frac{4y}{4} = \frac{-x}{4} + \frac{8}{4} \Rightarrow y = -\frac{1}{4}x + 2 \Rightarrow m_2 = -\frac{1}{4}$ ✓

$m_1 \cdot m_2 = 4 \cdot (-\frac{1}{4}) = -1$ ✓

perpendicular ✓ (2pts)

2. Determine the equation of the line passing through $\overset{x_1\ y_1}{(-3,7)}$ and $\overset{x_2\ y_2}{(-1,-5)}$. Show all work. Express your answer in slope-intercept form.

$m = \frac{y_2 - y_1}{x_2 - x_1} = \frac{-5-7}{-1-(-3)} = \frac{-12}{2} = -6$ ✓

$y - y_1 = m(x - x_1)$ ✓

$y - (-5) = -6(x - (-1))$

$y + 5 = -6(x + 1)$

$y = -6x - 6 - 5$

$y = -6x - 11$ ✓

Equation: $y = -6x - 11$ (3pts)

3. a. Find the radius of the circle that passes through the point $\overset{x\ y}{(3,7)}$ and has its center at $\overset{h\ k}{(-1,4)}$. Show all work.

$(x-h)^2 + (y-k)^2 = r^2$

$(3-(-1))^2 + (7-(4))^2 = r^2$

$16 + 9 = r^2$

$25 = r^2$

$5 = r$

Radius: 5 (1pt)

b. Find the equation of that circle. Express your answer in center-radius form.

Equation: $(x+1)^2 + (y-4)^2 = 25$ (2pts)

4. Solve analytically. Show all work. Express your answer in interval notation.

$2(x-4) < 3 - 5(2x+1)$

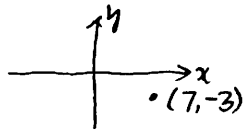
$2x - 8 < 3 - 10x - 5$ ✓

$2x - 8 < -2 - 10x$

$12x < 6$ ✓

$x < \frac{1}{2}$ ✓

$(-\infty, \frac{1}{2})$ ✓ (2pts)



5. Find the point that is symmetric to $(7,-3)$:

   a. With respect to the x-axis $(7,3)$ (1pt)

   b. With respect to the y-axis $(-7,-3)$ (1pt)

   c. With respect to the origin $(-7,3)$ (1pt)

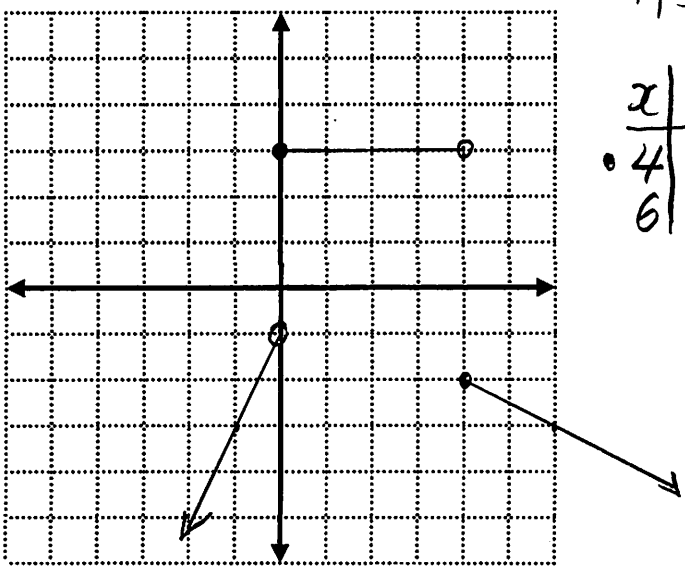
6. Graph the piecewise function. (3pts)

$$f(x)=\begin{cases} 2x-1 & \text{for } x<0 \\ 3 & \text{for } 0\le x<4 \\ -\frac{1}{2}x & \text{for } x\ge 4 \end{cases}$$

x	$f(x)=2x-1$, for $x<0$
0	-1
-1	-3

x	$f(x)=3$, for $0\le x<4$
0	3
4	3

x	$f(x)=-\frac{1}{2}x$, for $x\ge 4$
4	-2
6	-3



7. Given $f(x)=3x^2-6x+4$ and $g(x)=x^2-3x-10$, find $(f-g)(x)$ and state the domain of $(f-g)(x)$ in interval notation.

$$(f-g)(x)=f(x)-g(x)$$
$$=(3x^2-6x+4)-(x^2-3x-10)$$
$$=(3x^2-6x+4)+(-x^2+3x+10)$$
$$=2x^2-3x+14$$

$(f-g)(x)=2x^2-3x+14$ (2pts)

Domain of $(f-g)(x)$: $(-\infty,\infty)$ (1pt)

8. Given $f(x)=3x$ and $g(x)=2x^2-4x-7$, evaluate $(g\circ f)(x)$ and simplify.

$(g\circ f)(x) = g(f(x)) = g(3x)$

$= 2(3x)^2-4(3x)-7$

$= 2\cdot 9x^2-12x-7$

$= 18x^2-12x-7$

$18x^2-12x-7$ (2pts)

9. Determine the domain of the function. Express your answer in interval notation.

$f(x)=\sqrt{x+2}$

$x+2\geq 0$

$x\geq -2$

Domain: $[-2,\infty)$ (2pts)

10. Write an equation for a function that has the shape of $y=|x|$ that is shifted left 3 units, reflected about the x-axis, and shifted down 4 units.

$y=|x| \rightarrow y=|x+3| \rightarrow y=-|x+3| \rightarrow y=-|x+3|-4$ (3pts)

11. For the graph of $f(x)=-2x^2-24x-64$, state the coordinates of the vertex.

$-\frac{b}{2a} = -\frac{-24}{2(-2)} = -6$

$f(-6) = -2(-6)^2-24(-6)-64 = 8$

$(-6,8)$ (2pts)

12. Find the EXACT zeros of $f(x)=x^2-4x-41$ algebraically.

$x^2-4x-41=0$ ✓

$x^2-4x+4=41+4$ ✓

$(x-2)^2=45$ ✓

$x-2=\pm\sqrt{45}$ ✓

$x=2\pm\sqrt{45}=2\pm 3\sqrt{5}$ ✓ ✓

or $a=1,\ b=-4,\ c=-41$

$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$ ✓

$=\frac{-(-4)\pm\sqrt{(-4)^2-4\cdot 1\cdot(-41)}}{2(1)}$

$=\frac{4\pm\sqrt{16+164}}{2}$

$=\frac{4\pm\sqrt{180}}{2}$ ✓

$=\frac{4\pm 6\sqrt{5}}{2}=2\pm 3\sqrt{5}$ ✓

$2\pm 3\sqrt{5}$ (3pts)

13. Solve and write interval notation for the solution set: $|x+4|>5$

$x+4>5$ or $x+4<-5$ ✓

$x>1$ ✓ $x<-9$ ✓

$(-\infty,-9)\cup(1,\infty)$ ✓ ✓ (3pts)

14. Find the exact solution(s): $\sqrt{x+7}=x+1$

$x+7=(x+1)^2$ ✓

$x+7=x^2+2x+1$ ✓

$0=x^2+x-6$ ✓

$0=(x+3)(x-2)$ ✓

$x=-3$ or $x=2$ ✓

$\{2\}$ ✓ (3pts)

15. Find the exact solution(s): $\frac{2}{x+5}+\frac{1}{x-5}=\frac{16}{x^2-25}$ $\cdot(x+5)(x-5)$

$2(x-5)+1(x+5)=16$ ✓✓✓

$2x-10+x+5=16$ ✓

$3x-5=16$

$3x=21$

$x=7$ ✓

$\{7\}$ (3pts)

16. Determine the leading term, the leading coefficient, and the degree of the polynomial. Then classify the polynomial function as constant, linear, quadratic, cubic, or quartic.

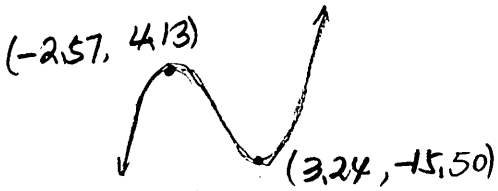
$f(x)=4x^3-7x^2+\frac{2}{3}x-6$

Leading term: $4x^3$ (1pt) Leading coefficient: 4 (1pt)

Degree of the polynomial: 3 (1pt) Classify the function: Cubic (1pt)

17. a. Graph the function using the given viewing window [-10, 10, -30, 20]. Determine all relative maxima and minima of the function. Round answers to two decimal places.

$f(x)=0.2x^3-0.2x^2-5x-4$



Maxima: 4.13 (1pt)

Minima: -15.50 (1pt)

b. Determine the interval(s) where $f(x)$ is increasing. Write your answer in interval notation.

Increasing: $(-\infty, -2.57) \cup (3.24, \infty)$ (1pt)

18. Data on airline revenue from add-on fees are listed in the following table. Use a graphing calculator to fit a regression line to the data, and let x=0 represent the year 2010.

Year, x	Airline Revenue from Add-On Fees (in billions), y
2010, 0	\$22.6
2011, 1	\$32.5
2012, 2	\$36.1
2013, 3	\$42.6
2014, 4	\$49.9

a) Linear Regression Equation:

$y = 6.47x + 23.8$ (2pts)

b) Predict airline revenue from add-on fees in 2025.

120.85

Revenue in 2025: $\approx$ \$120.9 billion (2pts)

19. The data in the following table shows healthcare costs in the U.S. between 1990 and 2013.

Year, x	Cost (per person), y
1990, 0	\$1,947
1996, 6	\$3,157
2002, 12	\$4,330
2007, 17	\$5,774
2013, 23	\$7,114

a) Using your graphing calculator, find the R^2 value for each model. Round answers to 4 decimal places, and let x= 0 represent year 1990. (2pts)

Linear: 0.9955

Quartic: 1

b) Based on the R^2 value, which function is the best fit? Quartic (1pt)

c) Using your graphing calculator, find the leading term for each model. Round answers to 4 decimal places.

Linear: $226.7737x$ (2pts)

Quartic: $-0.0599x^4$

d) Based on the end-behavior of each model, is the function you chose in part b) appropriate? Give a reason for your answer.

No. The end behavior of the graph is ↓↓. This implies that costs will drop in the future. But the healthcare costs would keep rising overtime. (2pts)

20. For the polynomial function $f(x)=x^4-2x^3+34x^2-98x-735$,

a) Find the zeros; that is, solve $f(x)=0$.

From the graph of f(x), -3 and 5 are rational zeros of f(x).

$$\begin{array}{r|rrrr} -3 & 1 & -2 & 34 & -98 & -735 \\ & & -3 & 15 & -147 & 735 \\ \hline & 1 & -5 & 49 & -245 & 0 \end{array}$$

$$\begin{array}{r|rrrr} 5 & 1 & -5 & 49 & -245 \\ & & 5 & 0 & 245 \\ \hline & 1 & 0 & 49 & 0 \end{array}$$

$x^2+49=0$

$x^2=-49$

$x=\pm 7i$

$-3, 5, 7i, -7i$ (4pts)

b) Factor $f(x)$ into linear factors.

$f(x)=(x+3)(x-5)(x-7i)(x+7i)$ (1pt)

21. For the function $f(x)=\dfrac{2x+9}{x-3}$, find each of the following. If it doesn't exist, then answer "none."

a. Domain in interval notation. $x\neq 3$ $(-\infty,3)\cup(3,\infty)$ (1pt)

b. Equation of the vertical asymptote: $x=3$ (1pt)

c. Equation of the horizontal asymptote: $y=2$ (1pt)

d. Equation of the oblique asymptote: none (1pt)

e. x-intercept(s) as ordered pairs: $2x+9=0$, $x=-\frac{9}{2}$ $\left(-\frac{9}{2},0\right)$ (1pt)

f. y-intercept as an ordered pair: $f(0)=\frac{9}{-3}=-3$ $(0,-3)$ (1pt)

22. Find the critical values and solve the inequality. Give the solution in interval notation.

$\dfrac{x-3}{x+2}\leq 0$

Critical values: $-2, 3$ ✓ ✓

	③	⓪	④
$(x-3)$	−	−	+
$(x+2)$	−	+	+
$\frac{x-3}{x+2}$	+	⊖	+

-2 3

✓ ✓

$(-2,3]$ (4pts)

23. Find $\log_3 18$ using the change-of-base formula and your calculator. Round to four decimal places.

$\log_3 18 = \frac{\log 18}{\log 3} \approx 2.6309$

2.6309 (2pts)

24. Solve the exponential equation algebraically. Write solution(s) in exact form.

$3^{4x+2} = 27$

$3^{4x+2} = 3^3$ ✓

$4x+2=3$ ✓

$4x=1$

$x=\frac{1}{4}$ ✓

$\{\frac{1}{4}\}$ (3pts)

25. Solve the logarithmic equations algebraically. Write solution(s) in exact form.

$\log_3(x+5)+\log_3(x-5)=2$

$\log_3(x+5)(x-5)=2$ ✓

$3^2=(x+5)(x-5)$ ✓

$9=x^2-25$ ✓

$34=x^2$

$x=\sqrt{34}$ or ~~$x=-\sqrt{34}$~~ ✓

$\{\sqrt{34}\}$ (4pts)

26. Jennifer recently graduated and landed a new job earning $34,000. Even though retirement is not in her immediate future, she remembers her math teacher stressing the benefits of investing over a long period of time. Jennifer decided to invest $3,400. Assuming that she earns 5% compounded quarterly, how much money will Jennifer have in her account upon her retirement 42 years later? $B = P\left(1+\frac{r}{n}\right)^{nt}$

$B = 3400\left(1+\frac{0.05}{4}\right)^{4(42)}$

$\approx \$27,405.91$

$\$27,405.91$ (4pts)

27. The population of Jackson, Missouri was approximately 9,500 in 1990 and the growth rate was 2.1% per year. When will the population of Jackson be 20,500? $P(t) = P_0 e^{kt}$

$P(t) = P_0 e^{kt}$

$20{,}500 = 9500 e^{0.021t}$ ✓

$\frac{205}{95} = e^{0.021t}$

$\ln\left(\frac{205}{95}\right) = \ln e^{0.021t}$ ✓

$\ln\left(\frac{205}{95}\right) = 0.021t$ ✓

$t = \frac{\ln\left(\frac{205}{95}\right)}{0.021} \approx 36.6$ yrs ✓

36.6 yrs after 1990 (4pts)

28. Solve the following nonlinear system, giving the solution(s) as ordered pair(s).

$\begin{cases} y = x^2 + 6x + 9 \\ x + y = 3 \Rightarrow y = 3 - x \end{cases}$

$3 - x = x^2 + 6x + 9$

$0 = x^2 + 7x + 6$ ✓

$0 = (x+1)(x+6)$ ✓

$x = -1$ or $x = -6$

If $x = -1$, $y = 3 - (-1) = 4$ $\quad (-1, 4)$

If $x = -6$, $y = 3 - (-6) = 9$ $\quad (-6, 9)$

(−1, 4), (−6, 9) (4pts)

29. Use two variables to solve this problem and show your work:
Kathy inherited $3,000 and invested it in two municipal bonds that pay 2% and 4% simple interest. The annual interest is $100. Find the amount invested at each rate.

Let x = amount invested at 2% and
y = amount invested at 4%. ✓

$\begin{cases} x + y = 3000 \Rightarrow x = 3000 - y \\ 0.02x + 0.04y = 100 \text{ ✓} \end{cases}$

$0.02(3000 - y) + 0.04y = 100$

$60 - 0.02y + 0.04y = 100$

$0.02y = 40$

$y = 2000$

$x = 3000 - 2000$

$x = 1000$

Amount at 2%: #1000 ✓

Amount at 4%: #2000 ✓ (4pts)

30. Suppose the following matrix equation is true. Find x and y.

$$\begin{bmatrix} 5 & \frac{x}{2} \\ 2y & -8 \end{bmatrix} = \begin{bmatrix} 5 & 3 \\ -2 & -8 \end{bmatrix}$$

$\frac{x}{2} = 3 \Rightarrow x = 6$

$2y = -2 \Rightarrow y = -1$

$x =$ 6 (1pt)

$y =$ −1 (1pt)