

Math 111 Final Exam Form B

Section _____ Date _____

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1. Find the exact solution(s): $\left(\frac{3}{x+2} + \frac{2}{x} = \frac{4x-4}{(x+2)(x-2)} \right) \cdot x(x+2)(x-2)$

$$(x-4)(x+2) = 0 \quad \checkmark$$

$$x = 4 \text{ or } x = -2 \quad \checkmark$$

$$3 \cdot x(x-2) + 2 \cdot (x+2)(x-2) = (4x-4) \cdot x \quad \checkmark$$

$$3x^2 - 6x + 2x^2 - 8 = 4x^2 - 4x \quad \checkmark$$

$$5x^2 - 6x - 8 = 4x^2 - 4x \quad \checkmark$$

$$x^2 - 2x - 8 = 0 \quad \checkmark$$

{ 4 } (4 Pts)

2. If $g(x) = \frac{x-3}{4-x}$ find a formula for $g^{-1}(x)$.

$$y = \frac{x-3}{4-x} \quad \checkmark$$

$$\left(x = \frac{y-3}{4-y} \right) \cdot (4-y) \quad \checkmark$$

$$x(4-y) = y-3 \quad \checkmark$$

$$4x - xy = y-3 \quad \checkmark$$

$$4x + 3 = y + xy \quad \checkmark$$

$$4x + 3 = y(1+x) \quad \checkmark$$

$$\frac{4x+3}{1+x} = y \quad \checkmark$$

$$g^{-1}(x) = \frac{4x+3}{1+x} \quad \checkmark$$

$g^{-1}(x) = \frac{4x+3}{1+x}$ (4 Pts)

3. The data in the following table shows healthcare costs in a country between 1987 and 2009.

Year, x	Cost (per person), y
1987, 0	\$1,958
1991, 4	\$2,946
1998, 11	\$4,387
2004, 17	\$6,199
2009, 22	\$7,949

a) Using your graphing calculator find the R^2 value for each model. Round to 4 decimal places and let $x=0$ represent Year 1987. (2pts)

Linear: 0.9894
 Quadratic: 0.9985
 Cubic: 0.9993
 Quartic: 1.0000

b) Based on the R^2 value, which function is the best fit? Quartic (1 Pt)

c) Is this function appropriate for future predictions? Why or why not? (2 Pts)

$$y = -0.0374315227x^4 + 1.852859559x^3 - 24.75804751x^2 + 318.7820545x + 1958$$

$\begin{cases} n=4 \text{ even} \\ a_n < 0 \end{cases} \Rightarrow \text{end behavior: } \downarrow \downarrow$

No. According to the end behavior, costs would drop in the future

4. Write an equation, in slope-intercept form, for the line containing the point $(-2, 5)$ and parallel to the line with the equation $4x + 2y = 3$.

$$\frac{2y}{2} = \frac{-4x+3}{2} \quad \checkmark$$

$$y = -2x + \frac{3}{2}$$

$$\Rightarrow m_1 = -2$$

$$\Rightarrow m_2 = -2$$

$$y - y_1 = m(x - x_1)$$

$$y - 5 = -2(x - (-2))$$

$$y - 5 = -2(x + 2)$$

$$y - 5 = -2x - 4$$

$$y = -2x + 1$$

$$\underline{y = -2x + 1} \quad (4 \text{ Pts})$$

5. The table below shows the number candles sold at a charity fundraiser for 4 successive years.

Year, x	Number of candles sold, y
0	81
1	70
2	59
3	52

- a) Using a graphing calculator model the data with a regression line. What is the equation? (1 Pt)

$$y = -9.8x + 80.2$$

- b) Using the regression line predict the number of candles that will be sold in year 5. (1 Pt)

$$31.2 \approx 31 \text{ candles}$$

- c) Do you think the regression line is a good model for the data? Does the correlation coefficient support your claim? (2 Pts)

Yes. $r = -0.9950392418$. $|r|$ is close to 1

6. Let $f(x) = -2x^2 + 8x - 3$. Find the vertex of $f(x)$.

$$-\frac{b}{2a} = -\frac{8}{2(-2)} = 2 \quad \checkmark$$

$$f(2) = -2(2)^2 + 8(2) - 3$$

$$= -8 + 16 - 3$$

$$= 8 - 3 = 5 \quad \checkmark$$

$$\underline{(2, 5)} \quad (2 \text{ Pts})$$

7. Write an equation for a function that has the same shape of $y = x^2$, but shifted right 6 units and up 2 units.

$$\underline{y = (x - 6)^2 + 2} \quad (2 \text{ Pts})$$

8. Find the exact solution(s):
- $\sqrt{x+7} - 1 = x$

$$(\sqrt{x+7})^2 = (x+1)^2 \quad \checkmark$$

$$x+7 = x^2 + 2x + 1 \quad \checkmark\checkmark$$

$$0 = x^2 + x - 6 \quad \checkmark$$

$$0 = (x+3)(x-2) \quad \checkmark$$

$$x = \cancel{-3} \text{ or } \boxed{x = 2} \quad \checkmark$$

2 (4 Pts)

9. For the function
- $f(x) = 2x^2 + 3x + 2$
- , construct and simplify the difference quotient
- $\frac{f(x+h) - f(x)}{h}$
- .

$$\frac{[2(x+h)^2 + 3(x+h) + 2] - [2x^2 + 3x + 2]}{h} \quad \checkmark\checkmark$$

$$= \frac{2(x^2 + 2xh + h^2) + 3x + 3h + 2 - 2x^2 - 3x - 2}{h} \quad \checkmark\checkmark$$

$$= \frac{\cancel{2x^2} + 4xh + 2h^2 + 3h - \cancel{2x^2}}{h} = \frac{4xh + 2h^2 + 3h}{h} = 4x + 2h + 3 \quad \underline{4x + 2h + 3} \quad (4 \text{ Pts})$$

10. Determine whether the graph of
- $y = x^4 - 2x^2 - |x|$
- is symmetric with respect to the origin. Justify your response using algebraic techniques.

Replacing x with $-x$ and y with $-y$:

$$-y = (-x)^4 - 2(-x)^2 - |-x| \quad \checkmark$$

$$-y = x^4 - 2x^2 - |x| \quad \checkmark$$

No. The resulting equation is not equivalent to the original equation.

_____ (3 Pts)

11. Solve the following inequality, graph its solution on a number line, and write the answer in interval notation.

$$|x-2| > 4$$

$$x-2 > 4 \text{ or } x-2 < -4 \quad \checkmark$$

$$x > 6 \quad x < -2$$

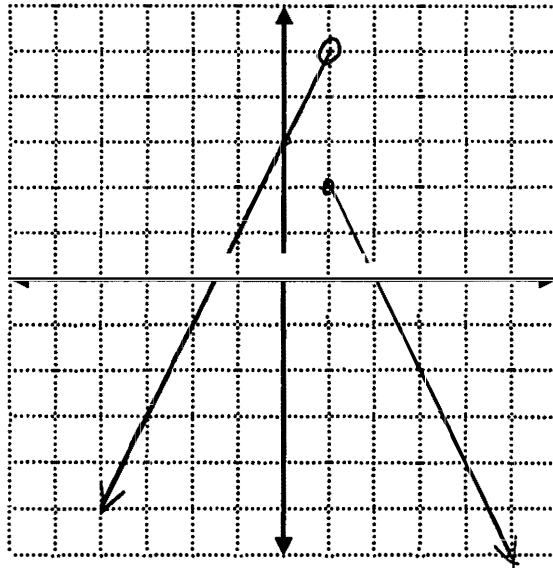


$(-\infty, -2) \cup (6, \infty)$ (4 Pts)

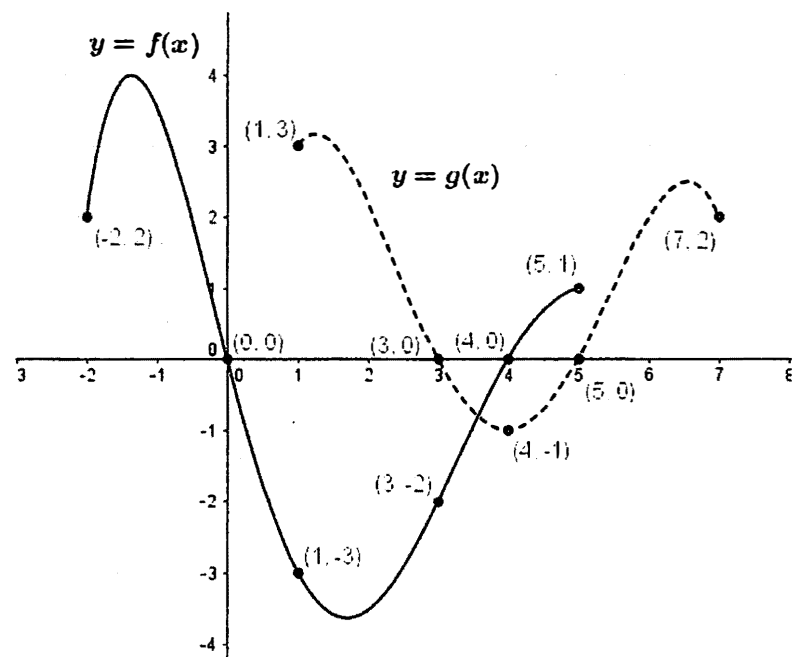
12. Graph $f(x) = \begin{cases} 2x + 3, & x < 1 \\ -2x + 4, & x \geq 1 \end{cases}$ (4 Pts)

$x \mid f(x) = 2x + 3, (x < 1)$
 $\begin{array}{r|l} 0 & 1 \mid 5 \\ & 0 \mid 3 \end{array}$

$x \mid f(x) = -2x + 4, (x \geq 1)$
 $\begin{array}{r|l} 0 & 1 \mid 2 \\ & 2 \mid 0 \end{array}$



13. Consider the graph of $y = f(x)$ and $y = g(x)$. Note: That graph of $y = g(x)$ is dashed.



a) What is the domain of $(f + g)(x)$? (2 Pts)

$[1, 5]$

b) What is the domain of $(\frac{f}{g})(x)$? (3 Pts)

$[1, 3) \cup (3, 5)$

14. Given that $\log_b 3 \approx 1.585$ and $\log_b 7 \approx 2.807$ estimate the value of $\log_b \frac{7}{3b}$.

$$\begin{aligned} \log_b \frac{7}{3b} &= \log_b 7 - \log_b 3 - \log_b b \quad \checkmark \checkmark \\ &= 2.807 - 1.585 - 1 \quad \checkmark \\ &= 0.222 \quad , \end{aligned}$$

0.222 (4 Pts)

15. Let $f(x) = x^3 - 5x^2 + 9x - 5$,

a) List all zeros of $f(x)$ both real and complex.

i) $\frac{p}{q} : \frac{\pm 1, \pm 5}{\pm 1}$

The possible rational zeros are $\pm 1, \pm 5$.

ii) From the graph of $f(x)$, 1 is a rational zero.

$$\begin{array}{r|rrrr} 1 & 1 & -5 & 9 & -5 \\ & & 1 & -4 & 5 \\ \hline & 1 & -4 & 5 & 0 \end{array} \quad \checkmark$$

$$x^2 - 4x + 5 = 0$$

$$\begin{aligned} x^2 - 4x + 4 &= -5 + 4 \quad \checkmark \\ (x-2)^2 &= -1 \\ x-2 &= \pm i \\ x &= 2 \pm i \end{aligned}$$

1, 2+i, 2-i (6 Pts)

b) Factor $f(x)$ completely. Write as a product of linear factors.

$f(x) = (x-1)(x-(2+i))(x-(2-i))$ (2 Pts)

16. Find the critical values and solve the inequality. Give the solution in interval notation.

$$\frac{2-x}{3x+4} \geq 0.$$

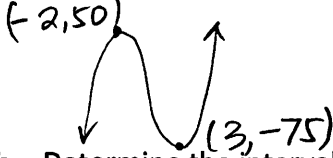
Critical values: $2, -\frac{4}{3}$ $\checkmark \checkmark$

	②	①	③
$(2-x)$	+	+	-
$(3x+4)$	-	+	+
$\frac{2-x}{3x+4}$	-	+	-
	$-\frac{4}{3}$	2	

$(-\frac{4}{3}, 2]$ (5 Pts)

17. Let $f(x) = 2x^3 - 3x^2 - 36x + 6$.

a. Locate all relative maxima on the graph of $y = f(x)$.



50 (2 Pts)

b. Determine the interval(s) where $f(x)$ is decreasing.

(-2, 3) (2 Pts)

18. Perform the indicated operation.

$$\frac{4+3i}{-2+5i} \cdot \frac{-2-5i}{-2-5i} = \frac{-8-26i+15}{4+25}$$

$$= \frac{-8-20i-6i-15i^2}{4-25i^2} = \frac{7-26i}{29} = \frac{7}{29} - \frac{26}{29}i$$

$\frac{7}{29} - \frac{26}{29}i$ (4 Pts)

19. Suppose Jacob and Emily both begin college at the same time. Jacob, however, is fortunate enough to complete his degree a year before Emily and secures a job with a salary of \$49,000. If Jacob invests 10% of his first year's salary in a retirement account bearing 8% compounded monthly over the next 37 years, how much more money to the nearest cent will he have as compared to Emily? Assume all variables are

the same except the extra year of savings. $B = P \left(1 + \frac{r}{n}\right)^m$

$$B = 49000 \times 0.1 \times \left(1 + \frac{0.08}{12}\right)^{12(37)}$$

$$\approx \$93,635.74$$

\$93,635.74 (4 Pts)

20. Solve algebraically for x , writing solution(s) in exact form.

a) $5^{4x-7} = 125$

$$5^{4x-7} = 5^3$$

$$4x-7 = 3$$

$$4x = 10$$

$$x = \frac{5}{2}$$

$\left\{\frac{5}{2}\right\}$ (3 Pts)

b) $\ln x - \ln(x-4) = \ln 3$

$$\ln\left(\frac{x}{x-4}\right) = \ln 3$$

$$\left(\frac{x}{x-4} = 3\right) \cdot (x-4)$$

$$x = 3(x-4)$$

$$x = 3x - 12$$

$$-2x = -12$$

$$x = 6$$

$\{6\}$ (4 Pts)

21. Given $f(x) = \frac{x^2 - x - 30}{x + 3} = \frac{(x-6)(x+5)}{x+3}$, determine the following, if it does not exist write "none":

a) the equation of the vertical asymptote.

$x = -3$ (1 Pt)

b) the equation of the horizontal asymptote.

none (1 Pt)

c) the equation of the oblique asymptote.

$y = x - 4$ (2 Pts)

$$\begin{array}{r|rr} -3 & -1 & -30 \\ & -3 & 12 \\ \hline & 1 & -4 & -18 \end{array}$$

d) the x -intercept(s). Write answer(s) as ordered pairs.

$(6, 0), (-5, 0)$ (2 Pts)

e) the y -intercept. Write answer(s) as an ordered pair.

$(0, -10)$ (2 Pts)

22. Solve the following nonlinear system, giving the solution(s) as ordered pair(s).

$$\begin{cases} x^2 + 2y^2 = 9 \\ x^2 - y^2 = 6 \end{cases} \quad \times (2)$$

$$\Rightarrow \begin{cases} x^2 + 2y^2 = 9 \\ + 2x^2 - 2y^2 = 12 \\ \hline 3x^2 = 21 \\ x^2 = 7 \\ x = \pm\sqrt{7} \end{cases}$$

$$\begin{aligned} 7 + 2y^2 &= 9 \\ 2y^2 &= 2 \\ y^2 &= 1 \\ y &= \pm 1 \end{aligned}$$

$(\sqrt{7}, 1), (\sqrt{7}, -1)$ (4 Pts)

$(-\sqrt{7}, 1), (-\sqrt{7}, -1)$

23. Suppose the following matrix equation is true.

$$\begin{bmatrix} 3 & -1 \\ x & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 5 \\ 16 \end{bmatrix}$$

a) Set up an equation that can be used to find the value of x.

$$\begin{array}{c} \begin{bmatrix} 3 & -1 \\ x & 4 \end{bmatrix} \begin{bmatrix} 2 \\ 1 \end{bmatrix} = \begin{bmatrix} 3(2) + (-1)(1) \\ x(2) + 4(1) \end{bmatrix} = \begin{bmatrix} 5 \\ 16 \end{bmatrix} \\ \begin{array}{cc} 2 \times 2 & 2 \times 1 \end{array} \end{array}$$

$$\underline{2x + 4 = 16} \quad (2 \text{ Pts})$$

b) Determine the value of x.

$$\begin{array}{l} 2x + 4 = 16 \\ 2x = 12 \\ x = 6 \end{array}$$

$$\underline{6} \quad (1 \text{ Pt})$$