The Application of the Theory of Power Law Distributions to U.S. Wealth Accumulation

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INTRODUCTION

In the recent literature on the dynamics of the distribution of wealth in the U.S. two empirical results have received considerable attention. First, that wealth is distributed according to a power law distribution at high wealth levels. Second, that wealth inequality has intensified over the past two decades (Weicher, 1997; Wolff, 1998; Rodriguez et al., 2002). Notwithstanding the focus on these issues, there is a lack of consensus surrounding the theoretical explanation of a power law distribution of wealth. Also, the choice of estimation technique of parameters for power law distributions is not a straightforward exercise (Brazauskas and Serfling, 2000).

In this paper we examine the robustness of the power law characterization of the wealth distribution at different wealth intervals based on wealth data obtained from the 2001 Survey of Consumer Finances. Household incomes are also fit to the power law model. In addition, we examine whether the exponent of the power law distribution displays an upward or downward pattern based on a comparison of the power law parameters obtained from the 1998 and 2001 Survey of Consumer Finances (SCF).

The aim of this paper is threefold: first, to determine the appropriateness of modeling SCF wealth data using a power law distribution, second, to test the hypothesis that there has been no change in the distribution of household wealth and third, to test the hypothesis that there is no difference in the distribution between financial wealth and nonfinancial wealth. These hypotheses are tested using data on household wealth obtained from the SCF for 1998 and 2001.

The main findings of the paper are as follows: (1) the power law characterization of wealth is appropriate with a lower bound wealth level of $100,000 for the 2001 data. (2) the power law exponent exhibits a downward tendency between 1998 and 2001, which is consistent with other indicators of increasing wealth inequality in the U.S. during the 1990s. (3) there is a statistical difference between the power law exponent for financial wealth and nonfinancial wealth.

The remainder of the paper is structured as follows. Section II provides a brief discussion of the dataset and empirical issues in the related literature as well as the approach used to obtain the parameters of the power law distribution. Section III discusses the empirical results.

DATA

The data used in this paper is obtained from the 1998 and 2001 Surveys of Consumer Finances for the United States. The SCF is known as a comprehensive source of household-level balance sheet, income, and socio-economic information for a representative sample[1] of the U.S. population. Since 1983, the Federal Reserve Board, in cooperation with the Statistics of Income Division of the Internal Revenue

[1]The database over-samples wealthy households in order to provide a larger basis for estimates of assets held by such households since they tend to underreport compared to other households. Sample weights are provided with the database to adjust each household to an estimate of its representation in the set of all U.S. households.

Service, has conducted the SCF every three years. A total of 4,309 and 4,449 households were interviewed in 1998 and 2001 respectively. (All dollar values were converted to 2001 dollars, using the CPI, for this study.) The wealth variable is defined as net worth, which is the difference between total asset holdings and total indebtedness. Since the net worth variable can have negative values, only households with non-negative wealth are included when fitting the power law distribution to the data.

Examination of summary statistics for household networth reported in the 1998 and 2001 SCF indicate coefficients of skewness that are positive and considerably above zero, indicative of a non-symmetric distribution with a long tail to the right. This is supported by mean to median ratios above one. In addition, the relatively high coefficient of kurtosis suggests an extremely peaked distribution. Histogram plots confirm positively skewed distributions of the wealth data. Testing the hypothesis that the power law exponent has declined between 1998 and 2001 provides the basis for determining whether there has been increased polarization of wealth in the U.S. during the late 1990s.

EMPIRICAL WEALTH DISTRIBUTION

In the context of disparate estimates of the power law exponent this paper explores the consistency of estimates over time. Specifically, this paper tests hypotheses highlighted in recent analyses of the U.S. wealth distribution. In null hypothesis form these are:

- That there is no difference in the U.S. wealth distribution across the 1998 and 2001 survey years.
- That there is no difference between power law exponents for financial and nonfinancial wealth.

THE POWER LAW DISTRIBUTION

The cumulative distribution function (CDF) for a random variable X is defined by $F(x) = P(X \le x)$. A random variable X is said to follow a power law (or Pareto) distribution if its CDF has the form $F(x) = 1 - \left(\frac{\beta}{x}\right)^{\alpha}$ where β and α are positive constants. For such a variable, the logarithm of $1 - F(x)$ is linear in the logarithm of x, since

$$\log(1 - F(x)) = \log(\left(\frac{\beta}{x}\right)^{\alpha}) = \alpha \log(\beta) - \alpha \log(x)$$

. From this it follows that for power law distributions, if $1 - F(x)$ and x are displayed on a log-log scale, the resulting graph will be linear.

Empirical estimates of the CDF for income and net worth are calculated for the 2001 SCF. These estimates are found by the formula $\overline{F}(x) = \frac{\sum_{i=1}^{n} w_i I(x_i \le x)}{\sum_{i=1}^{n} w_i}$ where n is the number of households, w_i is the weight of the i^{th} household, x_i is the variable of interest (income or net worth) and $I(condition)$ is an indicator function which returns a one if the condition is satisfied and a zero otherwise. Below are graphs of $1 - \overline{F}(x)$ for net worth and income on linear scales and then again on log-log scales. The linear scale plots are consistent with a power law distribution as they display an almost L shaped curve. If these quantities do result from an underlying power law distribution, the corresponding log-log plots should produce near linear plots. Both log-log plots appear to be rather linear. The linearity appears to increase as the income or net worth variables increase.

As we now have visual evidence in support of a power law model for these quantities, our new task is to obtain estimates of the α parameter. However, as Brazauskas and

Serfling (2000) indicate there are difficulties in estimating α. In particular, the standard Maximum Likelihood Estimator proves to be unsatisfactory. For purposes of this paper a least squares approach is used to estimate α. Thus, for a power law distribution with a known cutoff value of β dollars,

$$\log(1-F(x)) = \alpha\log(\beta) - \alpha\log(x) = \alpha\log\left(\frac{x}{\beta}\right)$$

. The α parameter can be estimated by fitting a linear model (with an intercept of 0) between $1-\bar{F}(x)$ and the transformed dollar values $\log\left(\frac{x}{\beta}\right)$.

Independent	R^2	d.f.	F	Sign.	α
Net Worth	.881	3620	26858	.000	.71
Income	.931	4099	55700	.000	1.08

RESULTS

Below are graphs and parameter estimates of the power law and the empirical CDF for both net worth and income (SCF 2001) with a cutoff dollar value of $\beta = \$10,000$. Note that in each case, the empirical estimates show too few households with small wealth and too many households with large wealth when compared to the power law estimate.

Independent	R^2	d.f.	F	Sign.	α
Income	.991	1483	160736	.000	1.6457

We can improve the power law fit with the empirical CDF by increasing the β cutoff level. For example, fitting the model to the 2001 income data with $\beta = \$100,000$ produces the following graph and parameter estimate. Note the increase in the R^2 value, indicating a tighter fit with the power law model.

An examination of the temporal behavior of the power law exponent for net worth is conducted using the two most recent Surveys of Consumer Finances (the 1998 SCF in 2001 dollars and the 2001 SCF datasets). The power law model is fit using a cutoff value of $\beta = \$100,000$. The results below indicate a statistically significant decline in the power law exponent from 1.162 to 1.110. This decrease corresponds to a thicker right tail and hence an increase in wealth inequality.

In order to explore the last hypothesis, the power law model was also fit to two forms of holding wealth, namely, financial assets and non-financial assets. A cutoff level of $\beta = \$50,000$ was utilized. The results are shown in the table below and based on these findings there is a statistically significant difference in the exponents for financial and nonfinancial wealth with financial assets showing a thicker right tail.

Net-worth	R^2	d.f.	F	Sign.	α	Std. Error.
1998	.980	2510	124334	.000	1.162	.003
2001	.966	2641	74492	.000	1.110	.004

Asset Type	R^2	d.f.	F	Sign.	α	Std. Error.
Financial	.954	2299	47733	.000	1.024	.005

Non-Financial	.965	3121	86394	.000	1.093	.004

CONCLUSION

The empirical findings indicate a change in the distribution of household wealth across the 1998 and 2001 Surveys of Consumer Finances. A second finding is that there is a statistical difference in the power law exponent for financial and nonfinancial wealth. The first finding is consistent with the theme of wealth polarization that can be found in much of the recent literature on household wealth distribution in the U.S. The second result suggests that financial wealth is more concentrated in the upper tail than nonfinanical wealth. These findings also provide the starting point for further research into the sources of wealth concentration at relatively high levels of wealth as well as likely explanations for the different power law exponents associated with financial and nonfinancial wealth.

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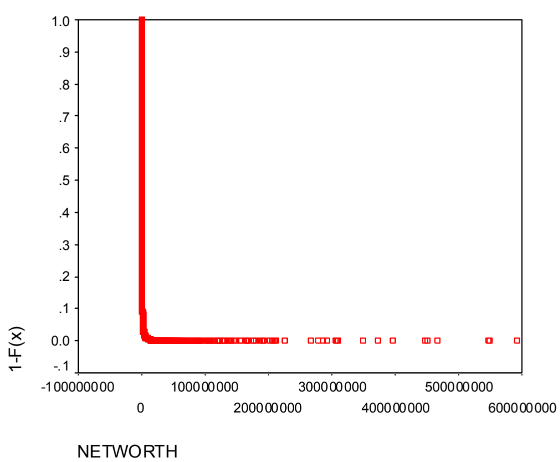
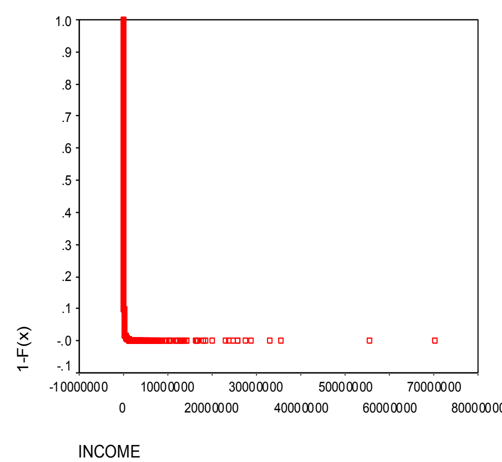
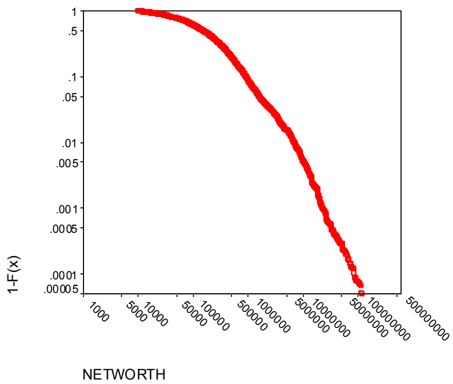
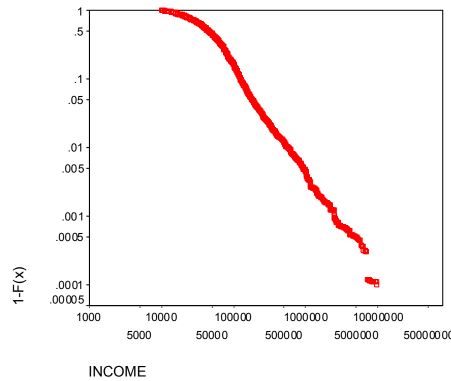
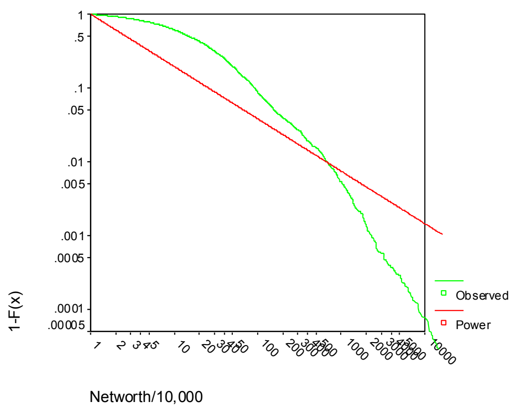
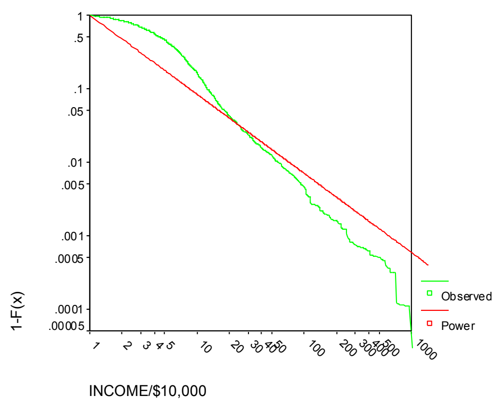


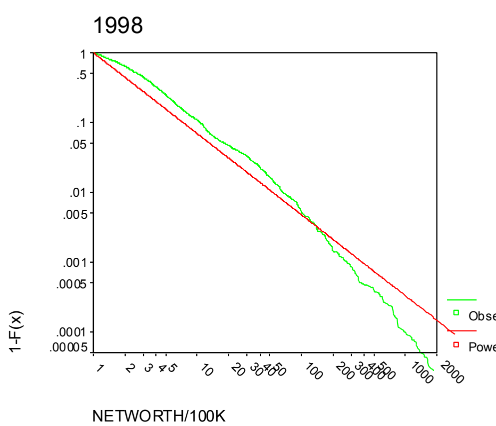


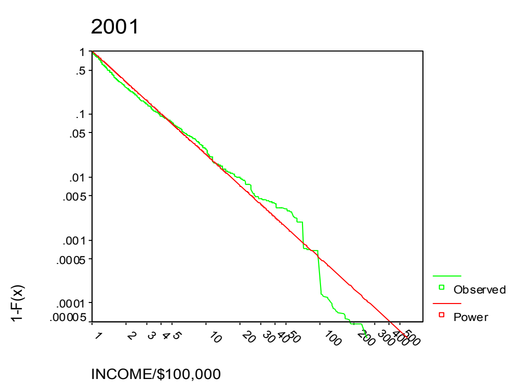

1-F(x)
NETWORTH

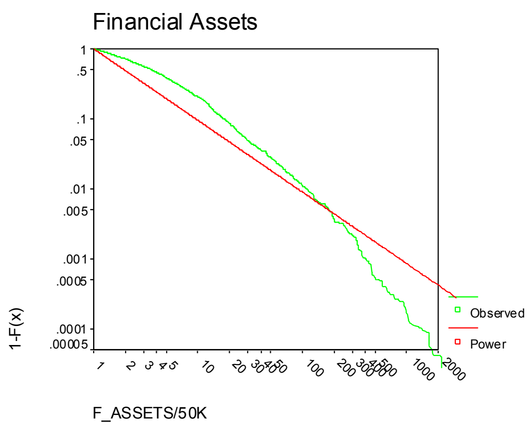

1-F(x)
INCOME

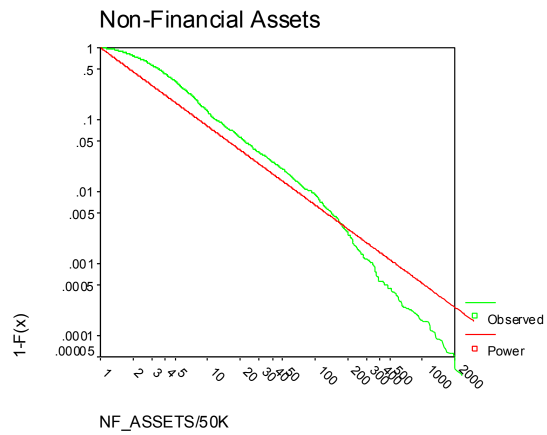

1-F(x)
Observed
Power
Networth/10,000


1-F(x)
Observed
Power
INCOME/$10,000


1998
1-F(x)
Observed
Power
NETWORTH/100K


2001
1-F(x)
Observed
Power
INCOME/$100,000


Financial Assets
1
.5
.1
.05
.01
.005
.001
.0005
.0001
.00005
1-F(x)
Observed
Power
1
2
3
4
5
10
20
30
40
50
100
200
300
400
500
1000
2000
F_ASSETS/50K


Non-Financial Assets
1
.5
.1
.05
.01
.005
.001
.0005
.0001
.00005
1-F(x)
Observed
Power
1
2
3
4
5
10
20
30
40
50
100
200
300
400
500
1000
2000
NF_ASSETS/50K